**Fall Sem. 2024-25**

**Tutorial- 5**

**Programme Name & Branch:** B. Tech.

**Course Name & Code:** Complex Variables and Linear Algebra (BMAT201L)

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**Practice all the questions**

1. Find the eigenvalues and the corresponding eigenvectors of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

   Answer: eigenvalues are $\lambda = 0, 3, 15$ and the corresponding eigenvectors are

   $X_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $X_2 = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$, $X_3 = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$.

2. Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{pmatrix}$ and hence find $A^{-1}$ if exists. Also find the matrix $B$ where $B = A^5 - 3A^4 - 7A^3 - A^2 + 2A + \mathrm{I}$

Answer: Characteristic equation of A is $\lambda^3 - 3\lambda^2 - 7\lambda - 1 = 0$. Clearly, $\mathrm{A}^3 - 3\mathrm{A}^2 - 7\mathrm{A} - \mathrm{I} = 0$ and $\mathrm{A}^{-1} = \begin{pmatrix} -6 & 2 & -3 \\ 7 & -2 & 4 \\ 3 & -1 & 1 \end{pmatrix}$. And $B = 2A + \mathrm{I}$.

3. Solve $x + 2y - 5z = -2,\ \ 2x - y + 3z = 4,\ \ x + 3y - z = 3$ using (i) Gaussian elimination and (ii) Gauss Jordan and Gauss Jordan methods.

   Answer: x=1, y=1, z=1