Reg. No:

Y/K/TX

VIT
Vellore Institute of Technology

Final Assessment Test – November 2024

Course: **BMAT205L - Discrete Mathematics and Graph Theory** Slot: **A1+TA1+TAA1**

Class NBR(s): **2587 / 2588 / 2589** Max. Marks: **100**

Time: **Three Hours**

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions

(10 X 10 = 100 Marks)

1. Determine the PCNF and PDNF of $(Q \vee (P \wedge R)) \wedge \neg((P \vee R) \wedge Q))$. **[10]**

2. Verify if $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$ using inference rules. Is the converse true? Justify. **[10]**

3. State and prove Lagrange's theorem. **[10]**

4. Let $H=\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ be a parity check matrix. Determine the (2, 5) group code function $e_H : B^2 \rightarrow B^5$. Create the decoding table. What is the original message if the received message is 11101. **[10]**

5. Use the method of generating function find a solution of the recurrence relation **[10]**

$a_n - 4a_{n-1} - 4a_{n-2} + 4^n = 0, n \geq 2, \text{given } a_0 = 2, a_1 = 8.$

6.a) Let X be any set and $\rho(X)$ be the power set of X. Verify if $< \rho(X), \leq >$ is a partially ordered set, where '$\leq$' denotes the relation ' is a subset of'. If $< \rho(X), \leq >$ is a lattice, then draw the Hasse diagram of $< \rho(X), \leq >$, where $X = \{1, 2, 3\}$. **[10]**

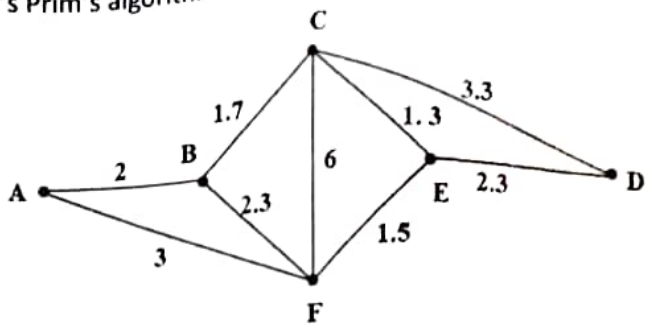
OR

6.b) Prove that in a lattice $< L, \leq >$, the distributive and modular inequalities are satisfied. **[10]**

7. Prove that any simple graph G with n – vertices and k – components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. **[10]**

8. Construct graphs for each one of the following **[10]**
   i. G is Euler but not Hamiltonian
   ii. Vertex connectivity of G = 3
   iii. Edge connectivity of G = 4
   iv. G has a Hamiltonian path
   v. G is a complete graph.

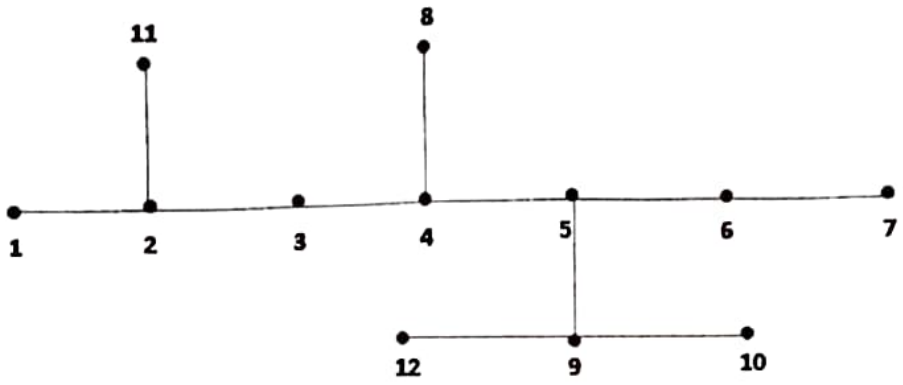
9.a) Determine and draw a minimum weighted spanning tree for the following graph using Kruskal's Prim's algorithm. [10]



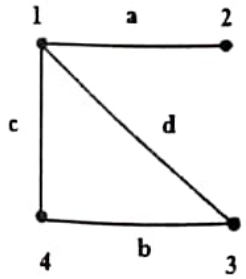
OR

9.b) i) Prove that a tree T with n vertices has n – 1 edges. [7]

ii) Determine the centre of the following tree T [3]



10. Given the following graph G, determine [10]



i. The chromatic polynomial of G using the iterative technique of edge addition and vertex merging.

ii. The chromatic number of G

iii. A chromatic partition of G.

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