VIT
Vellore Institute of Technology

Final Assessment Test – November/December 2

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2011 / 2012 / 2013 / 2014 / 2015 Slot: A2+TA

Time: Three Hours Max. Marks:

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MALPRAC

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Let the complex function $f(z)=\sqrt{|xy|}$. Is the function $f(z)$ satisfy the Cauchy Riemann equations at origin? If so, whether $f(z)$ is analytic at origin?

2. Determine the complex potential $f(z)=u+iv$ of an electric field in terms of z, given that $3u+2v=y^2-x^2+16xy$.

3. Find the image in the w-plane of the region bounded in the z-plane by the lines $x=1$, $y=1$ and $x+y=1$ under the square transformation $w=z^2$. Also, show the regions graphically.

4. Find the bilinear transformation w which maps the points $0,1,\infty$ in the z-plane into $-1,0,1$ in the w-plane. What are fixed points of z under w? Also find the region in the z-plane which maps to interior of the unit circle in the w-plane.

5. (i) Discuss the nature of the singularity for the function $f(z)=(z+1)\sin\left(\frac{1}{z-2}\right)$. [4]

   (ii) Find the Laurent's expansion of $f(z)=\frac{7z-2}{(z+1)z(z-2)}$ in the region $1<|z+1|<3$. [6]

6. Evaluate the real integral $\int_{-\infty}^{\infty}\frac{x^2+x+2}{(x^4+10x^2+9)}\,dx$ using contour integration.

7. (i) Let V be the vector space of polynomials of degree at most 2. Let $v_1=t^2+2t+1$, and $v_2=t^2+2$. Does the set $\{v_1,v_2\}$ span V? [5]

   (ii) Find a basis for the subspace W of R^3 spanned by $\{(1,2,2),(3,2,1),(11,10,7),(7,6,4)\}$. What is the dimension of W? [5]

8. Let $L:R^3\to R^3$ be defined by $L\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1&0&1\\1&1&2\\2&1&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}$.

   Find (i) basis for range L (ii) Kernel L and (iii) L^{-1} if exists.

9. The linear transformation $T:R^3\to R^3$ defined by $T(x,y,z)=(x+2y+z,x+y,x+3z)$.
   Let $\alpha=\{e_1=(1,0,0),e_2=(0,1,0),e_3=(0,0,1)\}$ and $\beta=\{(1,1,1),(1,1,0),(1,0,1)\}$ be the ordered bases for R^3. Find the basis change matrix from β to α.

10. Let W be the subspace of the Euclidean space R^3 with the standard inner product with the basis $S=\{(1,1,1),(0,1,1),(1,2,3)\}$. Find an orthonormal basis to S using Gram-Schmidt process.

11. Let $A=\begin{pmatrix}1&0&4\\0&4&0\\3&5&-3\end{pmatrix}$. Find (i) the eigenvector which corresponds to least eigenvalue of A and (ii) inverse of A using Cayley-Hamilton theorem.

12. Solve the following linear system using Gauss Jordan method:
    $2x-y+5z=3;\ x+2y+3z=12;\ 3x+3y+6z=21.$

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