Final Assessment Test (FAT) – May 2017

Course : **MAT1014 - Discrete Mathematics and Graph Theory**

Class NBR(s):**4425 / 4427 / 4429** Slot: **A2+TA2+TAA2**

Time: **Three Hours** Max. Marks: **100**

Answer any <u>FIVE</u> Questions

(5 X 20 = 100 Marks)

1. a) Obtain the principal conjunctive normal form of the formula S given by $(\sim P \to R) \wedge (Q \leftrightarrow P)$ and hence obtain Principal disjunctive normal form of S. **[10]**

   b) Show that the following sets of premises are inconsistent **[4+4+2]**
   (i) $A \to (B \to C), D \to (B \wedge \sim C), A \wedge D.$
   (ii) $P \to Q, P \to R, Q \to \sim R\ , P$
   Hence show that $A \to (B \to C), D \to (B \wedge \sim C), A \wedge D \Rightarrow P$

2. a) Prove that $(x)\left(P(x) \to \big(Q(y) \wedge R(x)\big)\right), (\exists x)(P(x)) \Rightarrow Q(y) \wedge (\exists x)(P(x) \wedge R(x)).$ **[8]**

   b) Show that the set of all the invertible elements of a monoid form a group under the same operation as that of the monoid. **[6]**

   c) Let $(S,*)$ be a given semigroup. Prove that there exists a homomorphism $f: S \longrightarrow S^S$, where $(S^S, \circ)$ is a semi group of functions from S to S under left composition. **[6]**

3. a) Show that if every element in a group is its own inverse, then the group must be abelian. **[4]**

   b) Prove that the order of a subgroup of a finite group divides the order of the group. **[10]**

   c) Let R be the set of real numbers in [0,1] and '≤' be the usual "less than or equal to" in R. Show that (R,≤) is a lattice. What are the operations of meet and join on this lattice? **[6]**

4. a) Prove that every chain is a distributive lattice. **[6]**

   b) Show that in a complemented distributive lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. **[10]**

   c) In any Boolean Algebra, show that $a \leq b \Rightarrow a + bc = b(a + c)$. **[4]**

5. a) Show that a mapping from one Boolean algebra to another which preserves the operations $\oplus\ and'$ also preserves the operation $*$. **[6]**

   b) Show that the Boolean expression **[4]**

$$\left[a * (b' \oplus c)\right]' * \left[b' \oplus (a * c')'\right]' = a * b * c'$$

   c) Prove that a connected graph G is Euler if and only if all vertices of G is even. **[10]**

6. a) Prove that a tree with n vertices has n-1 edges. **[10]**

   b) Prove that every circuit has even number of edges in common with any cut-set. **[10]**

7. a) Prove that a tree with two or more vertices is 2-chromatic. **[6]**

   b) Prove that the chromatic polynomial of a complete graph with n vertices is **[8]**

$$P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1)$$

   c) Show that if a bipartite graph has any circuits, they must all be of even length. **[6]**

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