Final Assessment Test (FAT) – May 2017

Course: **MAT1014 - Discrete Mathematics and Graph Theory**

Class NBR(s):4431 / 4434 /4436 / 4439 / 4444 / 4448 / 4934 Slot: **C1+TC1+TCC1**

Time: Three Hours Max. Marks: **100**

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) If *A* works hard, then either *B* or *C* will enjoy themselves. **[10]**
If *B* enjoys himself, then *A* will not work hard.
If *D* enjoys himself, then *C* will not.
Therefore, if *A* works hard, then *D* will not enjoy himself.
Show that the above statements constitute a valid argument by using the Rule CP.

b) Obtain the PCNF of the statement $S:(P\wedge Q)\vee(\neg P\wedge R)$. Using this obtain the PCNF of $\neg S$ and hence the PDNF of S. Also determine the unique representation of the PCNF and PDNF. **[10]**

2. a) (i) Prove that the set of idempotent elements of a commutative monoid $\langle M,*\rangle$ forms a submonoid. **[10]**

(ii) Show that the set $\mathbb{Q}^+$ of all positive rational numbers forms an abelian group under the operation $*$ defined by $a*b=\frac{1}{2}(a\cdot b);\forall a,b\in\mathbb{Q}^+$.

b) Find the code generated by the given parity matrix 'H' when the encoding function is $e:\mathrm{B}^3\to B^6$ **[10]**

$$H=\begin{bmatrix}1&1&0&1&0&0\\1&0&1&0&1&0\\0&1&1&0&0&1\end{bmatrix}$$

3. a) Let $(L,\le)$ be a lattice in which $*$, $\oplus$ denote the operations of meet and join respectively. For any $a,b\in L$, prove that $a\le b\Leftrightarrow a*b=a\Leftrightarrow a\oplus b=b$. **[10]**

b) Show that every chain is a distributive lattice. Also discuss about the converse of this statement with justification. **[10]**

4. a) State and prove the necessary and sufficient conditions for a connected graph to be an Euler graph. **[8]**

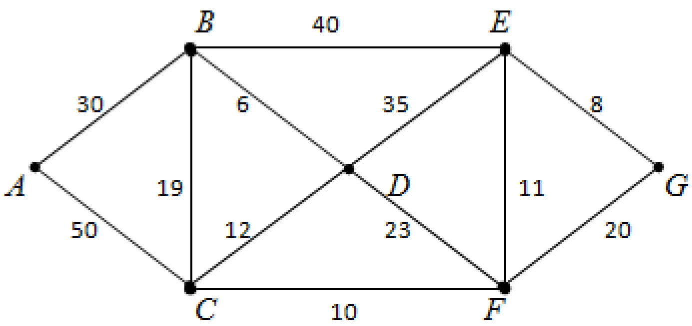
b) Obtain a graph G for the following adjacency matrix. **[6]**

$$A(G)=\begin{bmatrix}0&1&0&1&0&0\\1&0&0&1&1&1\\0&0&0&0&1&1\\1&1&0&0&1&0\\0&1&1&1&0&1\\0&1&1&0&1&0\end{bmatrix}.$$

Also find (I) the number of vertices in G.
(II) the number of edges in G.
(III) the degree of each vertex of G.
(IV) the number of loops in G and
(V) the number of components in G.

c) Let $\omega(G)$ be the number of components of G, then prove that the number of edges of a simple graph with ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$. **[6]**

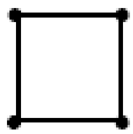
5. a) Use Kruskal's and Prim's algorithm to find a shortest path between vertices A to G in the following weighted network. **[10]**



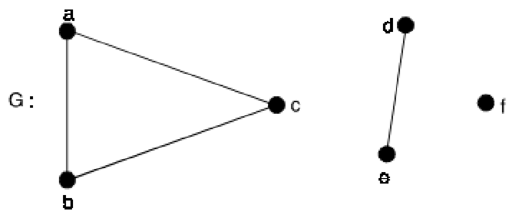
b) Prove that every tree is either unicentral or bicentral but not both. **[10]**

6. a) Find the chromatic polynomial and chromatic number for the following graph G. **[10]**

(i)



(ii)



b) Prove that a simple graph G on n vertices is a tree if and only if $P_n(\lambda) = \lambda(\lambda - 1)^{(n-1)}$ **[10]**

7. a) (i) Use Karnaugh map to simplify the following Boolean expression **[10]**

$w x \bar{y} \bar{z} + w \bar{x} y z + w \bar{x} y \bar{z} + w \bar{x} \bar{y} \bar{z} + \bar{w} x \bar{y} \bar{z} + \bar{w} \bar{x} y \bar{z} + \bar{w} \bar{x} \bar{y} \bar{z}$

(ii) Show that in a Boolean algebra $(a \oplus b') * (b \oplus c') * (c \oplus a') = (a' \oplus b) * (b' \oplus c) * (c' \oplus a)$.

b) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow ((\forall x)P(x)) \vee ((\exists x)Q(x))$ by using the indirect method of proof. **[10]**

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