

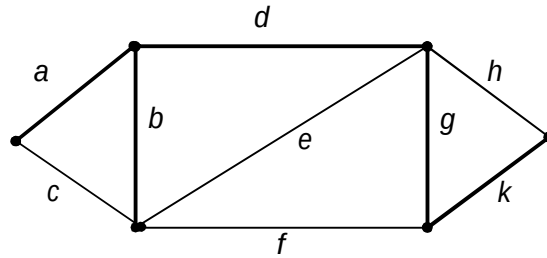


Answer any FIVE Questions

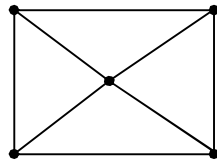
(5 X 20 = 100 Marks)

1. a) Obtain the pdnf and pcnf of the following formula and hence conclude whether it is a tautology [10]
 $(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$
- b) Test the consistency of the following Statements. [10]
 - i. If Jack studies well he will pass in exams.
 - ii. If Jack studies well he will get a job.
 - iii. Succeeding in exam and getting a job simultaneously are not possible for him.
 - iv. Jack either enjoys or studies well.
 - v. Finally, Jack enjoys.
2. a) Show that the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows from [10]
 - (i) $(x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$ and
 - (ii) $(\exists y)(M(y) \wedge \neg W(y))$
- b) Show that [10]
 - (i) $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$
 - (ii) $(P \vee Q) \wedge (\neg P \wedge (\neg P \wedge Q)) \Leftrightarrow (\neg P \wedge Q)$
3. a) (i) Prove that for any monoid $\langle M, * \rangle$, no two rows or columns of the composition table are identical. [4]
 (ii) Establish the isomorphism between the following two algebraic systems: [6]
 - I. $\langle F, \circ \rangle$ where $F = \{f^0, f^1, f^2, f^3\}$ with $f = f^1 = \{\langle 1,2 \rangle, \langle 2,3 \rangle, \langle 3,4 \rangle, \langle 4,1 \rangle\}$ and the composite functions are formed from the equation $f^k = f^{k-1} \circ f, k \geq 2$. Further, $f^0 = f^4$ is the identity map.
 - II. $\langle Z_4, +_4 \rangle$, the algebraic system of equivalence classes generated by congruence modulo 4 under addition modulo 4.
- b) State and prove Lagrange's theorem. [10]
4. a) What is the condition for a code to correct 'k' or fewer errors. Generate a single error correcting code with $m = 4$ and $n = 7$. [10]
- b) Obtain the Hasse diagrams of the lattices $\langle S_n, D \rangle$ when $n = 30, 45$. Which of these are complemented? Are these lattices distributive? Explain. [10]
5. a) (i) State and prove the isotonicity property of a lattice $\langle L, \leq \rangle$. [5]
 (ii) Obtain the simplified Boolean expression which is equivalent to the expression $m_0 + m_1 + m_2 + m_3$ [5]
- b) Obtain the Karnaugh map for the Boolean function $f = x_1 \circ [x_2 + (x_3 \circ x_4)]$. [10]
6. a) (i) Prove that any simple graph with n vertices has at most $\frac{n(n-1)}{2}$ edges. [5]
 (ii) Prove that the graph K_5 is nonplanar. [5]

- b) Determine all the fundamental circuits and fundamental cut-sets of the following graph with respect to the spanning tree shown by thick lines. Also find its vertex connectivity. **[10]**



7. a) Find the chromatic polynomial of the graph given below. **[10]**



- b) Determine a shortest spanning tree for the following graph using Kruskal's algorithm. Can we modify the algorithm to generate a Hamiltonian path. **[10]**

