



VIT[®]
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

**DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES**

Continuous Assessment Test – I (Re-CAT), September 2019

Course Code: MAT 3003

Date of Exam: 25-09-2019

Course Name: Complex Variables and Partial Differential Equations

Slot: A2+TA2+TAA2

Max. Marks: 50

Duration: 90 minutes

Answer ALL the questions

1. Find the analytic function $f(z) = u + iv$, given that $u - v = e^{-x}\{(x - y) \sin y - (x + y) \cos y\}$. [10]
2. An electrostatic field in the xy - plane is given by the potential function $u = 3x^2y - y^3$. Verify whether u is harmonic. Find the flux function and the complex potential. [10]
3. (a) If $w = u + iv = z^3$, prove that the level curves $u(x, y) = \alpha$, $v(x, y) = \beta$, cut each other orthogonally, where α and β are constants. [5]
(b) Find the image of the annulus $1 < |z| < 2$ under the transformation $w = \frac{z}{z - 1}$. [5]
4. Consider the points $1, i, -1$ on the z - plane mapped onto the points $-1, 0, 1$ on the w - plane under the bilinear transformation $f(z)$. [10]
(a) Determine the bilinear transformation $w = f(z)$.
(b) Find the image of the circle $|z| < 1$ under this transformation.
5. Find the Laurent's series expansion of $f(z) = \frac{1}{(z - 1)(z - 3)}$ in the region
(a) $1 < |z| < 3$ [5]
(b) $0 < |z - 1| < 2$ [5]
