**Vellore Institute of Technology**
(Deemed to be University under section 3 of UGC Act, 1956)

**Winter Semester 2019-2020**

**Continuous Assessment Test – I**

**Programme Name & Branch:** B. Tech.

**Course Name & Code: MAT 1014 Discrete Mathematics and Graph Theory** **Slot: A2+TA2+TAA2**

**Exam Duration:** 90 minutes **Maximum Marks:** 50

**Answer All the Questions (5 × 10 = 50)**

1. Find the PDNF and PCNF of $(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$.
   (i) With truth table method (ii) without truth table method.

2. (i) Prove that $\neg(p \wedge q) \rightarrow (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$.

   (ii) Test the validity of the following arguments:
   If milk is black then every crow is white.
   If every crow is white then it has 4 legs.
   If every crow has 4 legs then every Buffalo is white and brisk.
   The milk is black.
   So, every Buffalo is white.

3. (i) Show that the following statement is valid.
   All men are mortal
   Socrates is a man
   Therefore Socrates is a mortal.

   (ii) Prove that $(\exists x)(p(x) \wedge q(x)) \Rightarrow (\exists x)p(x) \wedge (\exists x)q(x)$.

4. Prove that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)(P(x)) \vee (\exists x)(Q(x))$ using indirect method.

5. Show that the set of all real $(2\times 2)$ matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}, ad - bc \neq 0$ is a group under matrix multiplication as binary operation.

# Answer key (MAT1014 A2 slot) Winter semester 2019-2020

1. PDNF: $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r)$

   PCNF: $(\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee q \vee r)$

2. (i)

$$\begin{aligned}
&\neg(p \wedge q) \to (\neg p \vee (\neg p \vee q)) \\
&\Leftrightarrow (p \wedge q) \vee (\neg p \vee (\neg p \vee q)) \\
&\Leftrightarrow (p \wedge q) \vee ((\neg p \vee \neg p) \vee q) \\
&\Leftrightarrow (p \wedge q) \vee (\neg p \vee q) \\
&\Leftrightarrow (p \vee (\neg p \vee q)) \wedge (q \vee (\neg p \vee q)) \\
&\Leftrightarrow ((p \vee \neg p) \vee q) \wedge ((q \vee q) \vee \neg p) \\
&\Leftrightarrow (T \vee q) \wedge (q \vee \neg p) \\
&\Leftrightarrow T \wedge (\neg p \vee q) \\
&\Leftrightarrow (\neg p \vee q)
\end{aligned}$$

2. (ii)

Let P : The milk is black

Q : Every crow is white

R : Every crow has four legs.

S : Every Buffalo is white

T : Every Buffalo is brisk

The given premises are

(i) $P \to Q$

(ii) $Q \to R$

(iii) $R \to S \wedge T$

(iv) P

The conclusion is S. The following steps checks the validity of argument.

| | | |
|---|---|---|
| 1. | $P \to Q$ | premise (1) |
| 2. | $Q \to R$ | Premise (2) |
| 3. | $P \to R$ | line 1. and 2. Hypothetical syllogism (H.S.) |
| 4. | $R \to S \wedge T$ | Premise (iii) |
| 5. | $P \to S \wedge T$ | Line 3. and 4.. H.S. |
| 6. | P | Premise (iv) |
| 7. | $S \wedge T$ | Line 5, 6 modus ponens |
| 8. | S | Line 7, simplification |

$\therefore$ The argument is valid

3.

(i) H(x): x is a man.

M(x): x is a mortal

S: Socrates

$$\forall(x)(H(x)\rightarrow M(x))\wedge H(S)\Rightarrow M(S)$$

(ii)

| | | |
|---|---|---|
| Step 1 | $(\exists x)(P(x)\wedge Q(x))$ | Rule P |
| Step 2 | $P(a)\wedge Q(a)$ | Rule ES |
| Step 3 | $P(a)$ | $\mathcal{I}_1$ |
| Step 4 | $Q(a)$ | $\mathcal{I}_1$ |
| Step 5 | $(\exists x)P(x)$ | $\{3\}$,EG |
| Step 6 | $(\exists x)Q(x)$ | $\{4\}$,EG |
| Step 7 | $(\exists x)P(x)\wedge(\exists x)Q(x)$ | $\{5,6\}$, $\mathcal{I}_3$ |

4.

| | | |
|---|---|---|
| Step 1 | $\neg((x)(P(x)\vee Q(x)))$ | Rule P |
| Step 2 | $\neg(x)P(x)\wedge\neg(\exists x)Q(x)$ | Rule T |
| Step 3 | $\neg(x)P(x)$ | $\mathcal{I}_1$ |
| Step 4 | $\neg(\exists x)Q(x)$ | $\mathcal{I}_1$ |
| Step 5 | $(\exists x)\neg P(x)$ | 3,Rule T |
| Step 6 | $(x)\neg Q(x)$ | 4,Rule T |
| Step 7 | $\neg P(a)$ | 5,ES |
| Step 8 | $\neg Q(a)$ | 6,US |

| | | |
|---|---|---|
| Step 9 | $\neg P(a)\wedge\neg Q(a)$ | $\{7,8\}$,$\mathcal{I}_3$ |
| Step 10 | $\neg(P(a)\vee Q(a))$ | Rule T |
| Step 11 | $(x)(P(x)\vee Q(x))$ | Rule P |
| Step 12 | $P(a)\vee Q(a)$ | US |
| Step 13 | $\neg(P(a)\vee Q(a))\wedge(P(a)\vee Q(a))$ | $\{10,12\}$, $\mathcal{I}_3$ |
| Step 14 | $F$ | Rule T |

5.

**Solution:** Let G be the set of all real (2×2) matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, ad-bc $\neq$0. For G to be group it should satisfies the following properties.

(1) Closure property: Let $\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ are arbitrary element of G, therefore, $(a_1d_1 - b_1c_1) \neq 0$ and $(a_2d_2 - b_2c_2) \neq 0$.

Now $\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix}$. As

$(a_1a_2 + b_1c_2)(c_1b_2 + d_1d_2) - (c_1a_2 + d_1c_2)(a_1b_2 + b_1d_2)$

$= (a_2d_2 - b_2c_2)\ (a_1d_1 - b_1c_1) \neq 0$. Hence $\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \in$G.

(2) Associative property: Let A=$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$, B=$\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ and C=$\begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}$,

then A(BC)

$$= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\left(\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}\right) = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\begin{bmatrix} a_2a_3 + b_2c_3 & a_2b_3 + b_2d_3 \\ c_2a_3 + d_2c_3 & c_2b_3 + d_2d_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1(a_2a_3 + b_2c_3) + b_1(c_2a_3 + d_2c_3) & a_1(a_2b_3 + b_2d_3) + b_1(c_2b_3 + d_2d_3) \\ c_1(a_2a_3 + b_2c_3) + d_1(c_2a_3 + d_2c_3) & c_1(a_2b_3 + b_2d_3) + d_1(c_2b_3 + d_2d_3) \end{bmatrix}$$

$$= \begin{bmatrix} a_1a_2a_3 + a_1b_2c_3 + b_1c_2a_3 + b_1d_2c_3 & a_1a_2b_3 + a_1b_2d_3 + b_1c_2b_3 + b_1d_2d_3 \\ c_1a_2a_3 + c_1b_2c_3 + d_1c_2a_3 + d_1d_2c_3 & c_1a_2b_3 + c_1b_2d_3 + d_1c_2b_3 + d_1d_2d_3 \end{bmatrix}$$

Similarly (AB)C

$$= \begin{bmatrix} a_1a_2a_3 + a_1b_2c_3 + b_1c_2a_3 + b_1d_2c_3 & a_1a_2b_3 + a_1b_2d_3 + b_1c_2b_3 + b_1d_2d_3 \\ c_1a_2a_3 + c_1b_2c_3 + d_1c_2a_3 + d_1d_2c_3 & c_1a_2b_3 + c_1b_2d_3 + d_1c_2b_3 + d_1d_2d_3 \end{bmatrix}$$

=A(BC).

(3) Identity element: As $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in$G such that

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \forall \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \text{G}.$$

(4) Existence of inverse: For $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in$G, we have

$$\begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \in \text{G such that}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

i.e. inverse of every element exist in G . Hence G is a group.