

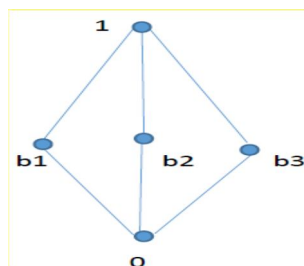


Answer any FIVE Questions

(5 X 20 = 100 Marks)



1. a) Show the following equivalent formula without constructing the truth table. [5]
 $(p \vee q) \vee (q \vee \neg r) \wedge (p \vee r) \Leftrightarrow p \vee q$
- b) Prove Demorgan laws in two variables and Compute equivalent of the following using Demorgan law [5]
 " It is not the case that roses are red and violets are blue "
- c) Explain PCNF and PDNF. Obtain PCNF and PDNF of the statement formula $(p \wedge q) \vee (\neg p \wedge q \wedge r)$. [10]
2. a) (i) Define tautology in statements. Show that SVR is tautologically implied from PVQ, $P \rightarrow R$ and $Q \rightarrow S$ [10]
 (ii) Prove the following by a counter example $(\forall x)\{p(x) \vee q(x)\} \rightarrow (\forall x)\{p(x)\} \vee (\forall x)\{q(x)\}$
 is not logically valid.
- b) Verify validity of the following arguments. [10]
 Every living thing is a plant or animal.
 John's gold fish alive and is not a plant.
 All animals have hearts.
 Therefore, John's gold fish has heart
3. a) Define a subgroup with an example. Prove that "A non-empty subset H of a group G is a subgroup of G if and only if [10]
 $a, b \in H \Rightarrow ab^{-1} \in H$
- b) Prove that "If G is a group in which $(ab)^k = a^k \cdot b^k$ for any three consecutive integers k, then G is abelian". [10]
4. a) Define Poset with an example. Suppose that $(S, \leq_1)$ and $(T, \leq_2)$ are two posets. Show that $(S \times T, \leq)$ is a poset, where $(s, t) \leq (u, v)$ if and only if $s \leq_1 u$ and $t \leq_2 v$. [10]
- b) (i) Define a Lattice. Let $S = \{a, b, c\}$. Show that the power set of S is a lattice under set inclusion and draw the corresponding Hasse Diagram. [10]
 (ii) Show that, in a distributive lattice the complement of any element is unique if it exists.



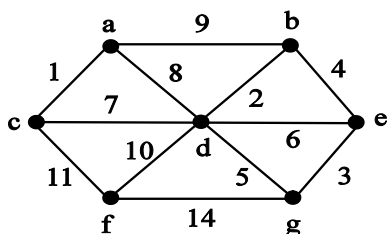
Show that the lattice in above Hasse diagram is not distributive using complements

5. a) Express $x_1 \oplus x_2$ as sum of products canonical and $x_1 * x_2$ as product sums canonical form . [10]
- b) Use k-maps to simplify the sum of the products canonical form [10]
 $\overline{w}x\overline{y}z + w\overline{x}yz + wxy\overline{z} + \overline{w}x\overline{y}z + \overline{w}xy\overline{z} + \overline{w}x\overline{y}z + \overline{w}xy\overline{z}$
 Verify your solution using the operations of Boolean algebra.

6. a) Show that in any graph $\sum_{v \in V} d(v) = 2\epsilon$, where ϵ is the total number of edges. Hence prove that [10]

$\delta \leq \frac{2\epsilon}{v} \leq \Delta$, Where δ and Δ are minimum and maximum degrees vertices, v is total no. of vertices in G.

- b) (i) Show that there are eleven non-isomorphic simple graphs on four vertices. [10]
(ii) Explain Eulerian and Hamiltonian Graphs with suitable examples.
7. a) Explain Prim's Algorithm and compute the minimum spanning tree of the following by using above algorithm. [10]



- b) Explain Chromatic number and Chromatic Polynomial with an example. [10]
Prove that the chromatic polynomial of a tree with n vertices is $\lambda(\lambda - 1)^{n-1}$.

