Winter Semester 2019-2020
Continuous Assessment Test – I
Programme Name: B. Tech.

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Course Name: Discrete Mathematics and Graph Theory **Course Code : MAT1014**
Slot : A1+TA1+TAA1 **Exam Duration: 90 minutes**

Answer All the Questions (5 × 10 = 50)

1. (a) Write down the contrapositive, the converse and the inverse of the statement, "If it is raining, then the home team wins." **(2)**

(b) Obtain PDNF and PCNF of the statement formula$\big(P \to (Q \wedge R)\big)\wedge((\neg P \to (\neg Q \wedge \neg R))$. **(8)**

2. Construct an argument to show that the following premises imply the conclusion "it rained."
(i) If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on.
(ii) If the sports day is held then then trophy will be awarded.
(iii) The trophy was not awarded. **(10)**

3. (a) Let $P(m, n)$ be "n is greater than or equal to m" where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of
(i) $(\exists n)(\forall m)P(m, n)$ (ii) $(\exists m)(\forall n)P(m, n)$. **(2)**
(b) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." **(8)**

4. Show that $(\exists x)(F(x) \wedge S(x)) \to (y)(M(y) \to W(y))$ and $(\exists y)(M(y) \to W(y))$ imply$(x)(F(x) \to \neg S(x))$. **(10)**

5. (a) Define Semigroup and Monoid. What is the relationship between them? Justify your answer. **(2)**
(b) Prove that the set of four functions f_1, f_2, f_3 and f_4 on the set of non-zero complex numbers $\mathbb{C} - \{0\}$ defined by
$f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}$, $\forall z \in \mathbb{C} - \{0\}$ forms an abelian group with respect to the composition of functions. **(8)**

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CAT-I - Answer Key

(1) (a) P: It is raining, Q: The home team wins

Given: $P \rightarrow Q$

Contrapositive: $\neg Q \rightarrow \neg P$ (ie) If the home team does not win, then it is not raining.

Converse: $Q \rightarrow P$ (ie) If the home team wins, then it is raining

Inverse: $\neg P \rightarrow \neg Q$ (ie) If it is not raining, then the home team does not win.

(b) PDNF: $(\neg P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge R)$

PCNF: $(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R)$.

(2) A: It rains, B: There is traffic dislocation

C: Sports day will be held, D: Cultural Programme will go on

E: Trophy will be awarded.

To prove: $\{(\neg A \vee \neg B) \rightarrow (C \wedge D),\ C \rightarrow E,\ \neg E\} \Rightarrow A$

Step	Statement	Rule
1	$C \rightarrow E$	P
2	$\neg E \rightarrow \neg C$	T, contrapositive of 1
3	$\neg E$	P
4	$\neg C$	{2,3} & Modus Ponens
5	$(\neg A \vee \neg B) \rightarrow (C \wedge D)$	P
6	$[\neg A \rightarrow (C \wedge D)] \wedge [\neg B \rightarrow (C \wedge D)]$	T, $(P \vee q) \rightarrow r \equiv (P \rightarrow r) \wedge (q \rightarrow r)$
7	$\neg A \rightarrow (C \wedge D)$	T, simplification
8	$\neg (C \wedge D) \rightarrow A$	T, contrapositive of 7
9	$\neg C \vee \neg D$	T, {4} & Addition

Step	Statement	Rule
10	$\neg(C \wedge D)$	T, {9} & De Morgan's law
11	A	{8, 10} & Modus Ponens

(3) (a) (i) False (ii) True

(b) Let $C(x)$ be " x is in this class "

$B(x)$ be " x has read the book "

$P(x)$ be " x passed the first exam ".

Then to prove

$$\{\exists x(C(x) \wedge \neg B(x)),\ \forall x(C(x) \rightarrow P(x))\} \Rightarrow \exists x(P(x) \wedge \neg B(x))$$

Step	Statement	Rule
1	$\exists x(C(x) \wedge \neg B(x))$	P
2	$C(a) \wedge \neg B(a)$	ES
3	$C(a)$	2 & simplification
4	$\forall x(C(x) \rightarrow P(x))$	P
5	$C(a) \rightarrow P(a)$	US
6	$P(a)$	{3, 5} & Modus Ponens
7	$\neg B(a)$	2, simplification
8	$P(a) \wedge \neg B(a)$	conjuction of 6 & 7
9	$\exists x(P(x) \wedge \neg B(x))$	EG.

4.

Step 1	$(\exists y)(M(y) \wedge \neg W(y))$	Rule P
Step 2	$(M(a) \wedge \neg W(a))$	ES
Step 3	$\neg(M(a) \longrightarrow W(a))$	Rule T
Step 4	$(\exists y)\neg(M(y) \longrightarrow W(y))$	EG
Step 5	$\neg(y)(M(y) \longrightarrow W(y))$	Rule T
Step 6	$(\exists x)(F(x) \wedge S(x)) \longrightarrow (y)(M(y) \longrightarrow W(y))$	Rule P
Step 7	$\neg(\exists x)(F(x) \wedge S(x))$	$\{5,6\}$, $\mathcal{I}_6$
Step 8	$(x)\neg(F(x) \wedge S(x))$	Rule T
Step 9	$\neg(F(a) \wedge S(a))$	US
Step 10	$F(a) \longrightarrow \neg S(a)$	Rule T
Step 11	$(x)(F(x) \longrightarrow \neg S(x))$	UG

5. (a)

Definition of semigroup and monoid. Every monoid is a semigroup but the converse need not be true. For example, $(\mathbb{N}, +)$ is a semigroup but not a monoid since identity element with respect to addition is not present, where $\mathbb{N}$ is a set of natural numbers.

(b)

Solution: Let $G = \{f_1, f_2, f_3, f_4\}$

$(f_1 \circ f_1)(z) = f_1(f_1(z)) = f_1(z)$

$\therefore f_1 \circ f_1 = f_1$

$f_2 \circ f_1 = f_2$, $f_3 \circ f_1 = f_3$, $f_4 \circ f_1 = f_4$

Again $(f_2 \circ f_2)(z) = f_2(f_2(z)) = f_2(-z) = -(-z) = z = f_1(z)$

$\therefore f_2 \circ f_2 = f_1$

Similarly $f_2 \circ f_3 = f_4$, $f_2 \circ f_4 = f_3$

$(f_3 \circ f_2)(z) = f_3(f_2(z)) = f_3(-z) = -\frac{1}{z} = f_4(z)$

$\therefore f_3 \circ f_2 = f_4$

Similarly $f_3 \circ f_3 = f_1$, $f_3 \circ f_4 = f_2$

$(f_4 \circ f_2)(z) = f_4(f_2(z)) = f_4(-z) = -\frac{1}{-z} = \frac{1}{z} = f_3(z)$

$\therefore f_4 \circ f_2 = f_3$

Similarly $f_4 \circ f_3 = f_2$, $f_4 \circ f_4 = f_1$

Using these results we have the composition table as given below :

$\circ$	f_1	f_2	f_3	f_4
f_1	f_1	f_2	f_3	f_4
f_2	f_2	f_1	f_4	f_3
f_3	f_3	f_4	f_1	f_2
f_4	f_4	f_3	f_2	f_1

From the table

(i) All the entries of the composition table are the elements of G.

$\therefore$ Closure axiom is true.

(ii) Composition of functions is in general associative.

(iii) Clearly f_1 is the identity element of G and satisfies the identity axiom.

(iv) From the table :

Inverse of f_1 is f_1 ; Inverse of f_2 is f_2

Inverse of f_3 is f_3 ; Inverse of f_4 is f_4

Inverse axiom is satisfied. (G, o) is a group.

(v) From the table the commutative property is also true.

$\therefore$ (G, o) is an abelian group.