



**DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES**

Continuous Assessment Test – I, January 2020

Course Code: MAT 3003

Date of Exam: 19-01-2020

Course Name: Complex Variables and Partial Differential Equations

Slot: A1+TA1+TAA1

Max. Marks: 50

Duration: 90 minutes

Answer ALL the questions

1. Find the analytic function $f(z) = u + iv$, given that $u = e^{-x} \cos y + xy$. Also show that the level curves $u(x, y) = \alpha, v(x, y) = \beta$ cut orthogonally. [10]
2. Discuss the transformation $f(z) = z^2$ as the flow of fluid around a corner. Draw the equipotential lines and the stream lines. Also find the complex velocity and speed of the flow. [10]
3. (a) Show that under the mapping $w = 1/z$, all circles and straight lines in the z -plane are transformed to circles and straight lines in the w -plane. [5]
(b) Under the mapping $w = e^{-z}$, find the image in the w -plane of the rectangle $R : 0 \leq x \leq 1, 0 \leq y \leq \pi/4$ in the z -plane. Is the mapping conformal? [5]
4. Determine the bilinear transformation that maps the points $z = 0, 1, \infty$ into the points $w = -i, 1, i$ respectively. Find the invariant points of this transformation. Find the image of $|z| < 1$ under this transformation. [10]
5. (a) Find the Taylor's series expansion of $f(z) = \frac{1}{4-3z}$ about the point $z_0 = 1 + i$. Determine the region of convergence. [4]
(b) Expand the function $f(z) = \frac{1}{(z-1)^2(z-3)}$ in Laurent series valid in the regions: [6]
(i) $0 < |z-1| < 2$ (ii) $|z| > 3$.
