VIT
Vellore Institute of Technology
Vellore – 632014, Tamil Nadu, India
DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST – I

Programme Name & Branch : B. Tech
Course Code : BMAT201L
Course Name : Complex Variables and Linear Algebra
Slot : B2+TB2+TBB2
Date of the Examination : 29-08-2022
Duration : 90 minutes **Max. Marks : 50**

General instruction(s): Answer All The Questions

Q. No	Question	Marks	Course Outcome (CO)	Bloom's Taxonomy (BL)
1.	Construct an analytic function whose real part is $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$.	10	CO1	BL2
2.	In the two dimensional fluid flow, the stream function is given by $\psi(x, y) = e^{x^2-y^2}\sin 2xy$. Find the complex potential function $f(z) = \phi + i\psi$ and hence find the velocity potential function $\phi(x, y)$.	10	CO1	BL3
3.	Find the image of the region bounded by the lines $x = 2$, $x = 4$ in the z-plane under the transformation $w = z^2$.	10	CO2	BL3
4.	Determine the bilinear transformation that maps the points $1-2i$, $2+i$, $2+3i$ in z-plane into the points $2+2i$, $1+3i$, 4 respectively in w-plane.	10	CO2	BL3
5.	Expand $f(z) = \dfrac{1}{z^2 - z - 6}$ as a Taylor's series about $z = 1$ and $z = -1$.	10	CO3	BL1

BMAT201L

CAT1 - KEY - B2 + TB2 + TBB2

(1Q) Given $u = \dfrac{\sin 2x}{\cosh 2y - \cos 2x}$

Let $f(z) = u + iv$

$$f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$f'(z) = \frac{2\cos 2x \cosh 2y - 2}{(\cosh 2y - \cos 2x)^2} + i\,\frac{2\sin 2x \sinh 2y}{(\cosh 2y - \cos 2x)^2}$$

By Milne-Thomson's method, we express $f'(z)$ in terms of z by putting $x = z$ and $y = 0$.

$$\therefore f'(z) = \frac{2\cos 2z - 2}{(1 - \cos 2z)^2} = \frac{-2}{2\sin^2 z}$$

$$f'(z) = -\operatorname{cosec}^2 z.$$

$$\therefore f(z) = \cot z + iC$$

(2Q) Given $\psi = e^{x^2-y^2} \sin 2xy$ is stream function and $f(z) = \phi + i\psi$.

~~By Milne-Thomson's method,~~

velocity potential $\phi = e^{x^2-y^2} \cos 2xy + C$

potential function $f = \phi + i\psi$.

$\cancel{= e^{x^2-y^2} \sin 2xy + i e^{x^2-y^2}}$

$$f = e^{x^2-y^2} \cos 2xy + i\, e^{x^2-y^2} \sin 2xy + C$$

$$= e^{x^2-y^2} (\cos 2xy + i \sin 2xy) + C$$

$$f = e^{x^2-y^2} \cdot e^{i2xy} + C$$

(3Q) Given $w = z^2$

$u + iv = x^2 - y^2 + i(2xy)$

Here $u = x^2 - y^2$, $v = 2xy$

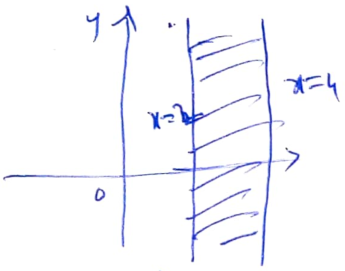
$x = 2 \Rightarrow u = 4 - y^2$, $v = 4y$

$\therefore y = \frac{v}{4} \Rightarrow u = 4 - \frac{v^2}{16}$ wich is a parabola

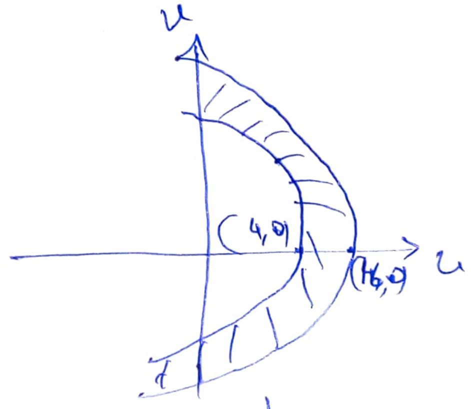
$x = 4 \Rightarrow u = 16 - y^2$, $v = 8y$

$\therefore y = \frac{v}{8}$

$\Rightarrow u = 16 - \frac{v^2}{64}$. which is also a parabola.



z-plane



w-plane

(4Q) Given ~~$z_1 = 2+2i$, $z_2 = 2i$~~

$z_1 = 1-2i$, $z_2 = 2+i$, $z_3 = 2+3i$, $z_4 = z$

$w_1 = 2+2i$, $w_2 = 1+3i$, $w_3 = 4$, $w_4 = w$.

we know that bilinear transformation preserve's cross ratio.

(i.e) $$\frac{(z_1 - z_2)(z_3 - z_4)}{(z_2 - z_3)(z_4 - z_1)} = \frac{(w_1 - w_2)(w_3 - w_4)}{(w_2 - w_3)(w_4 - w_1)}$$

By substituting given points, we get

$$w = \frac{-(32+12i)z + (20+18i)}{-(11-3i)z + (29+17i)}$$ which is the required bilinear transformation.

5a) Given $f(z) = \frac{1}{z^2-z-6}$

$$= \frac{1}{(z-3)(z+2)} = \frac{A}{(z-3)} + \frac{B}{(z+2)}$$

$$= \frac{1}{5}\left(\frac{1}{z-3} - \frac{1}{z+2}\right)$$

Taylor's series expansion of $f(z)$

(i) At $z=-1$

$$f(z) = -\frac{1}{5}\sum_{n=0}^{\infty}(-1)^n (z+1)^n - \frac{1}{20}\sum_{n=0}^{\infty}\left(\frac{z+1}{4}\right)^n, \quad |z+1|<1 \text{ and } |z+1|<4.$$

(ii) At $z=+1$,

$$f(z) = -\frac{1}{5}\sum_{n=0}^{\infty}\frac{(z-1)^n}{3} - \frac{1}{10}\sum_{n=0}^{\infty}\frac{(z-1)^m}{2^n}, \quad |z-1|<2 \text{ and } |z-1|<3.$$