Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Vellore – 632014, Tamil Nadu, India
DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST – I

Programme Name & Branch : B.Tech
Course Code : BMAT205L
Course Name : Discrete Mathematics and Graph Theory
Slot : A1+TA1+TAA1
Date of the Examination : 28.08.2022
Duration : 90 minutes **Max. Marks : 50**

General instruction(s): ANSWER ALL QUESTIONS

Q. No	**Question**	Marks	Course Outcome (CO)	Bloom's Taxonomy (BL)
1.	Find the principal conjunctive normal form and principal disjunctive normal form of $S \Leftrightarrow (P \rightarrow \neg Q) \wedge (Q \rightarrow R) \wedge \neg(P \wedge \neg R) \wedge (P \vee \neg R)$ and hence find whether S is a Tautology or Contradiction?	10	CO1	L3
2.	Determine the validity of the following argument: For students to do well in a discrete mathematics course, it is necessary that they study hard. Students who do well in courses do not skip classes. Therefore students who do well in a discrete mathematics course do not skip classes	10	CO1	L4
3.	(i) Negate the following statements: (a) Some integers are natural number (b) Every city in Canada is clean (ii) Demonstrate the following implication $\neg(x)\big(P(x) \wedge Q(x)\big),\ (x)P(x) \Rightarrow \neg Q(x)$	10	CO1	L2
4.	Write down the composition tables for $(Z_7, +_7)$ and $(Z_7^*, \times_7)$ where $Z_7^* = Z_7 - \{[0]\}$ and verify whether they are groups.	10	CO2	L3
5.	State and prove the necessary and sufficient condition for a nonempty subset H of G to be a subgroup of the group $(G,*)$	10	CO2	L4

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VIT

School Of Advanced Sciences

Department Of Mathematics

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BMAT205L - DMGT - Fall 22-23

Answer key

1. $S \Leftrightarrow (\neg P \vee \neg Q) \wedge (\neg Q \vee R) \wedge (\neg P \vee R) \wedge (P \vee \neg R)$

$\Leftrightarrow (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee R)$

$\wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \cancel{\neg} Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R)$

$S \Leftrightarrow (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge$

$(P \vee \neg Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R)$

which is required pcnf. of s

$\neg S \Leftrightarrow (\neg P \vee Q \vee \neg R) \wedge (P \vee Q \vee R) \cancel{\wedge (P \vee \neg Q \vee R)}$

$\neg\neg S \Leftrightarrow S \Leftrightarrow (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$ - Pdnf of S.

Since both pcnf and pdnf exist, S is neither Tautology nor contradiction.

2.

P: students do well in discrete Mathematics course

Q: Students study hard R: students do well in course

$\neg S$: Students do not skip classes.

Premises: $P \rightarrow Q$, $R \rightarrow \neg S$.

conclusion: $P \rightarrow \neg S$.

(1) $P \rightarrow Q$ Rule P

(2) $R \rightarrow \neg S$ Rule P

conclusion cannot be derived using the premises given.

3)

(a) $(\exists x)(P(x) \wedge Q(x))$

$\neg(\exists x)(P(x) \wedge Q(x)) \Leftrightarrow (x)(P(x) \rightarrow \neg Q(x))$

Ans: All integers are not natural numbers

3) i) b) Ans: Some city in Canada is not clean

ii) (1) $\neg(x)(P(x) \wedge Q(x))$ Rule P

(2) $(\exists x)\neg(P(x) \wedge Q(x))$ Rule T (1)

(3) $(\exists x)(\neg P(x) \vee \neg Q(x))$ Rule T (2) De Morgan

(4) $(\exists x)(P(x) \rightarrow \neg Q(x))$ Rule T (3)

(5) $P(x) \rightarrow \neg Q(x)$ Rule ES (4)

(6) $(x)P(x)$ Rule P

(7) $P(x)$ Rule US (6)

(8) $\neg Q(x)$ Rule T (7, 5)

4)

$+_7$	[0]	[1]	[2]	[3]	[4]	[5]	[6]
[0]	0	1	2	3	4	5	6
[1]	1	2	3	4	5	6	0
[2]	2	3	4	5	6	0	1
[3]	3	4	5	6	0	1	2
[4]	4	5	6	0	1	2	3
[5]	5	6	0	1	2	3	4
[6]	6	0	1	2	3	4	5

$\times_7$	[1]	[2]	[3]	[4]	[5]	[6]
[1]	1	2	3	4	5	6
[2]	2	4	6	1	3	5
[3]	3	6	2	5	1	4
[4]	4	1	5	2	6	3
[5]	5	3	1	6	4	2
[6]	6	5	4	3	2	1

4) $(Z_7, +_7)$

(i) Since all the entries belong to Z_7, closure property holds

(ii) $([a] +_7 [b]) +_7 [c] = [a] +_7 ([b] +_7 [c])$

$\forall [a], [b], [c] \in Z_7$

(iii) $[0]$ is the identity

(iv) Inverse $[0]^{-1} = [0]$, $[1]^{-1} = [6]$

$[2]^{-1} = [5]$, $[3]^{-1} = [4]$, $[4]^{-1} = [3]$

$[5]^{-1} = [2]$, $[6]^{-1} = [1]$

$\therefore (Z_7, +_7)$ is a group

$(Z_7^*, \times_7)$

i) Closure property holds

ii) $([a] \times_7 [b]) \times_7 [c] = [a] \times_7 ([b] \times_7 [c])$

iii) $[1]$ is the identity

iv) $[1]^{-1} = [1]$, $[2]^{-1} = [4]$, $[3]^{-1} = [5]$

$[4]^{-1} = [2]$, $[5]^{-1} = [3]$, $[6]^{-1} = [6]$

$\therefore (Z_7^*, \times_7)$ is a group.

5) Proof:

If G is a group, H is non empty subset of $(G, *)$

Suppose H is a subgroup

Let $a, b \in H \Rightarrow a, b^{-1} \in H \Rightarrow a * b^{-1} \in H$.

$\therefore a, b \in H \Rightarrow a * b^{-1} \in H$

Now suppose $a, b \in H \Rightarrow a * b^{-1} \in H$. — ①

i) $a * a^{-1} \in H \Rightarrow e \in H$.

ii) $e, a \in H \Rightarrow e * a^{-1} \in H \Rightarrow a^{-1} \in H$.

iii) Let $a, b \in H \Rightarrow a, b^{-1} \in H$ [from (ii)]

$\Rightarrow a * (b^{-1})^{-1} \in H$ by def ①

$\Rightarrow a * b \in H$. $\therefore$ Closure holds

iv) H is a subset of G. Associative is true.

$\therefore$ H is a subgroup.