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Vellore Institute of Technology

Final Assessment Test – November 2022

Course: BMAT203L - Linear Algebra and Differential Equations
Class NBR(s): 0817 / 0819
Time: Three Hours
Slot: C2+TC2+TCC2
Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

CALCULATOR IS PERMITTED

Answer any TEN Questions
(10 X 10 = 100 Marks)

1. Solve the following system of equations by using Gauss Elimination method:

$2U + 5V + 2W - 3X = 3$

$3U + 6V + 5W + 2X = 2$

$4U + 5V + 14W + 14X = 11$

$5U + 10V + 8W + 4X = 4$

2. If $\lambda = 15$ is an eigen-value of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, find the other two eigen values of A and also find the eigen values of $A^3 - 5A^2 + 18I$.

3. Find the values of x for which the series $\sum_{n=0}^{\infty} \frac{(2x+3)^n}{5^{n+1}}$ converges. Also, find the radius of convergence and interval of convergence of it.

4. Test the absolute convergence of the series

$$\frac{2}{3} - \frac{2 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 5 \cdot 8}{3 \cdot 5 \cdot 7} - \frac{2 \cdot 5 \cdot 8 \cdot 11}{3 \cdot 5 \cdot 7 \cdot 9} + \cdots.$$

5. Find a power series of $f(x) = \tan^{-1}(x^2)$ and compute the sum $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{(2n+1)/2}\,(2n+1)}$.

6. Find the Taylor series for the function $f(x) = x^3 \log(x)$ about $x = 1$ upto fourth order.

7. i) Find the equilibrium solutions of

$$\frac{dy}{dx} = 5y^2 - 6y - y^3.$$

ii) Classify the equilibrium solutions as sink, source and node.

iii) Sketch the slope field of it.

iv) Draw the phase line of it.

v) Sketch the solution graph of it.

8. Solve the initial value problem that models the amount of salt in the pond:

$$\frac{dS}{dt} = \frac{5}{2} - \frac{26S}{200 - t} \; ; \quad S(0) = 0.$$

9. Solve the Van der Pol equation using Euler's method.

10. Solve analytically the following completely decoupled system of equations

$$\frac{dx}{dt} = -2x \; ; \; \frac{dy}{dt} = -y \; ; \; (x(0), y(0)) = (1,1).$$

11. Solve the linear system $\frac{dY}{dt} = \begin{pmatrix} 0 & 2 \\ -3 & 2 \end{pmatrix} Y$ with initial condition $Y_0 = (1,1)$ using analytic techniques and discuss the nature of the solution.

12. Using Qualitative techniques, discuss the behaviour of the Damped Harmonic Oscillator with sinusoidal forcing given by $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \sin t.$

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