



# VIT

Vellore Institute of Technology  
(Deemed to be University under Section 3 of U.G. Act, 1956)

## SCHOOL OF ADVANCED SCIENCES

Winter Semester 2022-2023

### Continuous Assessment Test –II

Programme Name & Branch : B.Tech

Slot: D2+TD2 (Common)

Course Name & code: Probability and Statistics BMAT202L

Exam Duration: 90 Min.

Maximum

Marks: 50

General instruction(s): Answer ALL Questions

(Table or Charts are to be permitted)

Q.No.	Question	Max Marks	CO	BL																														
1.	<p>A researcher is studying the relationship between sleep duration (in hours) and academic performance (GPA) for a group of 20 students. They collect the following data:</p> <table border="1"><thead><tr><th>Student</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th></tr></thead><tbody><tr><td>Sleep duration</td><td>4</td><td>7</td><td>8</td><td>9</td><td>6</td><td>6.5</td><td>7.5</td><td>4.5</td><td>8.5</td></tr><tr><td>GPA</td><td>8</td><td>9</td><td>9.5</td><td>8.5</td><td>8</td><td>7</td><td>5</td><td>6</td><td>6.5</td></tr></tbody></table> <p>Are the sleep duration and GPA of the students consistent?</p>	Student	1	2	3	4	5	6	7	8	9	Sleep duration	4	7	8	9	6	6.5	7.5	4.5	8.5	GPA	8	9	9.5	8.5	8	7	5	6	6.5	10	CO3	BL5
Student	1	2	3	4	5	6	7	8	9																									
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2.	<p>(a) A machine produces identical units. The proportion of defective units produced by the machine is known to be <math>1/20</math>. We also know that successive units are statistically independent. Obtain the probability that in a sample of 10 units, there are at most 2 defectives.</p> <p>(b) The probability that a person recovers from a serious</p>	10	CO3	BL3																														

	disease is 0.40. Find the probability that at least one of the 8 persons admitted to a hospital will survive.			
3.	<p>The height of female students at a University follows approximately a normal distribution, with mean 60 inches and standard deviation 2. Find the probability that a female student selected at random has height</p> <p>a) less than 58 inches</p> <p>b) between 58 inches and 62 inches.</p>	10	CO3	BL3
4.	<p>It is seen that 17.26% people lost their money in the stock market. In a certain year 640 people invested in a stock and 63 persons lost their money. Can the stock be considered as believable at 1% level of significance (LOS)? What about at 5% LOS?</p>	10	CO4	BL3
5.	<p>A bakery claims their new muffins have an average diameter of 5 centimetres (cm) with a standard deviation of 0.5 cm. You suspect the muffins might be smaller. To investigate this, you randomly sample 49 muffins and measure their diameters. The average diameter of your sample is 4.8 cm. Determine if there's enough evidence to suggest the bakery's claim (average diameter of 5 cm) is inaccurate at a 5% significance level. What about 1% level of significance?</p>	10	CO4	BL4

Ans-1:

Student	1	2	3	4	5	6	7	8	9	
Sleep (x) duration	4	7	8	9	6	6.5	7.5	4.5	8.5	
$R_x$	9	5	3	1	7	6	4	8	2	→ 1 marks
GPA	8	9	9.5	8.5	8	7	5	6	6.5	
$R_y$	4.5	2	1	3	4.5	6	9	8	7	→ 2 marks
$d = R_x - R_y$	4.5	3	2	-2	2.5	0	-5	0	-5	
$d^2$	20.25	9	4	4	6.25	0	25	0	25	Σ d <sup>2</sup> = 93.5

Rank correlation = 0.225 (4 marks)

Hence there is a positive correlation <sup>between</sup> the sleep duration and GPA of the students.

Note-1: There is nothing wrong in this question. Even the class size is 20, we have only data of 9 students available. Hence the rank correlation will be calculated only for these 9 students.

Note-2: If the students <sup>calculated</sup> ~~used~~ the correlation coefficient and concluded then also it can be acceptable.

Ans of Q2: (a) use of Binomial distribution,

$$p = \frac{1}{20}, q = \frac{19}{20}, P(X=0) = {}^n C_r p^r q^{n-r} \rightarrow 1 \text{ mark}$$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.99 \rightarrow 4 \text{ marks}$$

$$(b) P(X \geq 1) = 1 - P(X=0) = 1 - {}^8 C_0 (0.4)^0 \cdot (0.6)^8 = 0.983 \rightarrow 5 \text{ marks}$$

Ans of Q3: (a)  $P(X < 58) = P\left(\frac{X-60}{2} < \frac{58-60}{2}\right)$

$$= 0.1587 \quad 5 \text{ marks}$$

$$(b) P(58 < X < 62) = 0.6826 \quad 5 \text{ marks}$$

Ans of Q4:

$H_0$ : the stock is believable

$H_1$ : the stock is not believable

$$Z = \frac{P - P_0}{\sqrt{\frac{P_0 Q_0}{n}}}, \quad P = \frac{63}{640} = 0.0984$$

5 marks

$$P = 0.1726, \quad Q = 0.8274$$

$$Z = -4.96, \quad |Z| = 4.96$$

$$|Z_{\alpha}| = 2.33 \text{ at } 1\% \text{ LOS}, \quad |Z_{\alpha}| = 1.645 \text{ at } 5\% \text{ LOS}$$

$H_0$ : rejected 1% LOS

$H_0$ : rejected 5% LOS

5 marks

Q5:

$H_0$ :  $\bar{x} = 4$

$H_1$ :  $\bar{x} \neq 4$

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{4.2 - 5}{0.5 / \sqrt{49}} = -2.8$$

5 marks

$$|Z| = 2.8$$

$|Z_{\alpha}|$  at 1% LOS 2.33 and 5% is 1.645

5 marks

↓  
 $H_0$  rejected

↓  
 $H_0$  rejected