Final Assessment Test – Jan/Feb 2023

VIT
Vellore Institute of Technology

Course: **BMAT101L - Calculus**

Class NBR(s): **5431 / 6121 / 6433**

Slot: **C1+TC1**

Time: **Three Hours**

Max. Marks: **100**

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Sketch the graph of the function $f(x)=4x^3+3x^2-6x+1$ using the following steps
   a) Identify where the extrema of $f(x)$ occur.
   b) Find the intervals where the function is increasing and decreasing.
   c) Find the point(s) of inflection.
   d) Find where the graph of $f(x)$ concave up and concave down.

2. a) Find the area of the region enclosed by the parabolas $y=x^2$ and $y=2x-x^2$.

   b) Find the volume of the solid generated by revolving the region between the y - axis and the curve $xy=2,\ 1\le y\le 4$, about y - axis.

3. a) Show that $u=\cos(x+at)+\sin(x-at)$ satisfies the *Wave equation* $a^2\dfrac{\partial^2 u}{\partial x^2}=\dfrac{\partial^2 u}{\partial t^2}$. **[5]**

   b) If $u=x+3y^2-z^3;\ v=4x^2yz;\ w=2z^2-xy$, then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,1,1)$. **[5]**

4. Expand $f(x,y)=\sin(xy)$ in powers of $(x-1)$ and $\left(y-\dfrac{\pi}{2}\right)$ using Taylor's theorem (up to second order).

5. Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

6. a) Sketch the region R in the xy plane bounded by $y=x^2, x=2, y=1$.

   b) Evaluate the double integral $I=\iint_R \left(x^2+y^2\right)\,dx\,dy$.

   c) Evaluate the same double integral by using change of order of integration.

7. Evaluate $\iiint_V \dfrac{dx\,dy\,dz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2+y^2+z^2=a^2$.

8. a) Evaluate $\int_0^\infty e^{-x^2}dx$. **[5]**

   b) Find the volume of an ellipsoidal shell described by $\left(\dfrac{x}{a}\right)^2+\left(\dfrac{y}{b}\right)^2+\left(\dfrac{z}{c}\right)^2=1$ in the first octant using Gamma function. **[5]**

9. a) Find the unit normal to the surface $xy^3z^2 = 4$ at $(-1,-1,2)$. [5]

b) Find the directional derivative of the function $\phi(x,y,z) = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of $4\vec{i} - 2\vec{j} + \vec{k}$. [5]

10. If $\vec{F} = \left(x^2y^3 - z^4\right)\vec{i} + 4x^5y^2z\vec{j} - y^4z^6\vec{k}$, find (a) curl $\vec{F}$ (b) div $\vec{F}$ (c) div (curl $\vec{F}$).

11. Show that the line integral $\int_{(1,-2,1)}^{(3,1,4)} \left(2xy + z^3\right)dx + x^2dy + 3xz^2dz$ is independent of the path and hence evaluate $\int_{(1,-2,1)}^{(3,1,4)} \bar{F}.d\bar{r}$.

12. Verify the Gauss - Divergence Theorem, if $\vec{F} = 2xz\vec{i} + yz\vec{j} + z^2\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = 4$.

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