VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

School of Advanced Sciences
Department of Mathematics
BMAT201L – Complex Variables and Linear Algebra
Question Pattern and Model Paper for Practice Only

MODULE	Total Number of Questions
1. Analytic Functions	Two
2. Conformal mapping and Bilinear Transformation	Two
3. Complex Integration	Two
4. Vector space	One
5. Linear Transformation	Two
6. Inner product spaces	One
7. Matrices and System of Equations	Two

No	**ANSWER ANY 10 Questions (10 x 10 = 100) out of 12 Questions**
1	An electrostatic field in the xy-plane is given by the potential function $\phi = x^2 - y^2$, find the stream function and complex potential function.
2	Examine whether the following function can be the real part of an analytic function $f(z)$. If so, find its harmonic conjugate $v \equiv v(x,y)$. Then write the analytic function $f = u + iv$ as a function of the complex variable z: where suppose u = $e^{2xy}\sin(x^2 - y^2)$
3	Determine the image of $\|z-2\| = 2$ under the transformation $w = z^2$ and sketch the graph of the region in $z-$ plane and its image in the $w-$ plane.

4	Find the bilinear transformation $w = f(z)$ that maps one triad (z_1, z_2, z_3) onto another triad (w_1, w_2, w_3) Also, determine the derivative, and the invariant points of the points $-i$, 1 and i onto -1, 0 and 1
5	(i) Expand the function $f(z) = sin(z)$ in a Taylor's series at $z = \pi$. (ii) Find the Laurent's series of $f(z) = \frac{z^2-1}{z^2+5z+6}$ for (a) $\lvert z\rvert < 2$, (b) $\lvert z\rvert > 3$ and (c) $2 < \lvert z\rvert < 3$.
6	Evaluate the complex integral $\int_c \frac{z-1}{(z+1)^2(z-2)} dz$, where C is the circle $\lvert z - i \rvert = 2$.
7	Find the basis and dimension of the subspaces $U = \{(x, y, z, w, t) / 2x - y - z = 0 = w - 3t\}$ $W = \{(x, y, z, w, t) / z + w = 0\}$ Also determine the basis for U+W.
8	Determine (i) ker(T) (ii) nullity(T) (iii) range(T) (iv) rank(T) for the following linear transformations without using the dimension theorem: $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(\mathbf{v}) = \mathbf{A}\mathbf{v}$ where $\mathbf{A} = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 4 & 12 & 20 \end{pmatrix}$
9	Suppose that $T : R^3 \to R^3$ is a linear transformation so that $T\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, T\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}, T\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 8 \end{bmatrix}$ then find $T\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and determine $T\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.
10	Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace U of $\mathbb{R}^4$ spanned by v_1=(2,2,2,2), v_2=(2,4,8,10), v_3=(2,−6,−8,−4)
11	Show that the following equations are consistent and solve them. $x + y + z = 6$ $x + 2y + 3z = 14$ $x + 4y + 7z = 30$
12	Verify Cayley-Hamilton theorem for A and hence find the inverse of A from this theorem. $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$

****