VIT

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

WINTER SEMESTER 2022-2023

SCHOOL OF ADVANCED SCIENCES

DEPARTMENT OF MATHEMATICS

CONTINUOUS ASSESSMENT TEST – II

Course Code	: BMAT202L
Course Name	: Probability and Statistics
Slot	: F1+TF1
Duration	: **90 Minutes**
Date	: **17.03.2023**

Max. Marks: 50

Answer ALL the following questions.

1. Find the equations of the regression lines from the following data. Also estimate the value of Y when $X = 71$ and the value of X when $Y = 70$.

(10M)

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

2. Fit a Poisson distribution for the following distribution (10M)

x:	0	1	2	3	4	5	6
f:	314	335	204	86	29	9	3

3. In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class. (Assume normal distribution of marks)

(10M)

4. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumers of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is significant decrease in the consumption of tea after the increase in duty.

(10M)

5. A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D. of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D. of 6.3 cm. Do the data indicate that Americans are, on the average, taller than the Englishmen?

(10M)

Answer Key

Continuous Assessment Test II

BMAT202L Probability and Statistics

F1+TF1

1. $\bar{X} = 68$ $\bar{Y} = 69$
   Regression line Y on X is $Y = 0.666X + 25.3$
   Y=72.58
   Regression line X on Y is X=0.5455Y+30.36
   X=68.53

2.

   $\lambda = 1.2$

x	0	1	2	3	4	5	6
f	295	354	213	85	26	6	1

3.

Let X represent the percentage of marks scored by the students in the examination.

Let X follow the distribution $N(\mu, \sigma)$.

Given: $P(X < 45) = 0.10$ and $P(X > 75) = 0.05$

i.e., $$P\left(-\infty < \frac{X-u}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.10 \text{ and}$$

$$P\left(\frac{75-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \infty\right) = 0.05$$

i.e., $$P\left(-\infty < Z < \frac{45-\mu}{\sigma}\right) = 0.10 \text{ and}$$

$$P\left(\frac{75-\mu}{\sigma} < Z < \infty\right) = 0.05$$

$\therefore$ $$P\left(0 < Z < \frac{\mu-45}{\sigma}\right) = 0.40 \text{ and}$$

$$P\left(0 < Z < \frac{75-\mu}{\sigma}\right) = 0.45$$

From the table of areas, we get

$$\frac{\mu-45}{\sigma} = 1.28 \text{ and } \frac{75-\mu}{\sigma} = 1.64$$

i.e., $\mu - 1.28\,\sigma = 45$ (1)

and $\mu + 1.64\,\sigma = 75$ (2)

Solving equations (1) and (2), we get

$\mu = 58.15$ and $\sigma = 10.28$

Now P (a student gets first class)

$$= P(60 < X < 75)$$

$$= P\left\{\frac{60-58.15}{10.28} < Z < \frac{75-58.15}{10.28}\right\}$$

$$= P\{0.18 < Z < 1.64\}$$

$$= P\{0 < Z < 1.64\} - P\{0 < Z < 0.18\}$$

$$= 0.4495 - 0.0714 = 0.3781$$

$\therefore$ Percentage of students getting first class = 38 (approximately)

Now percentage of students getting second class

= 100 – (sum of the percentages of students who have failed, got first class and got distinction)

= 100 – (10 + 38 + 5), approximately.

= 47 (approximately)

4.

Let p_1 and p_2 be the proportions of the consumers before and after the increase in duty respectively.

Then $$p_1 = \frac{800}{1000} = \frac{4}{5} \quad \text{and} \quad p_2 = \frac{800}{1200} = \frac{2}{3}.$$

$$H_0 : p_1 = p_2$$
$$H_1 : p_1 > p_2$$

One-tailed (right-tailed) test is to be used. Let LOS be 1%. $\therefore z_\alpha = 2.33$.

$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad \text{where } P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$= \frac{800 + 800}{2200} = 0.7273$$

$$= \frac{0.8 - 0.67}{\sqrt{0.7273 \times 0.2727 \times \left(\frac{1}{1000} + \frac{1}{1200}\right)}}$$

$$= \frac{0.13 \times \sqrt{1000 \times 1200}}{\sqrt{0.7273 \times 0.2727 \times 2200}} = 6.82$$

Now $|z| > z_\alpha$.

$\therefore$ The difference between p_1 and p_2 is significant at 1% level.

i.e. H_0 is rejected and H_1 is accepted.

i.e. there is significant decrease in the consumption of tea after the increase in duty.

5.

$$n_1 = 6400, \ \bar{x}_1 = 170 \quad \text{and} \quad s_1 = 6.4$$
$$n_2 = 1600, \ \bar{x}_2 = 172 \quad \text{and} \quad s_2 = 6.3$$

$$H_0 : \mu_1 = \mu_2 \quad \text{or} \quad \bar{x}_1 = \bar{x}_2,$$

i.e. the samples have been drawn from two different populations with the same mean.

$$H_1 : \bar{x}_1 < \bar{x}_2 \quad \text{or} \quad \mu_1 < \mu_2.$$

Left-tailed test is to be used.

Let LOS be 1%. $\therefore z_\alpha = -2.33$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

[$\because \sigma_1 \simeq s_1$ and $\sigma_2 \simeq s_2$. Refer to Note 2 under Test 4]

$$= \frac{170 - 172}{\sqrt{\frac{(6.4)^2}{6400} + \frac{(6.3)^2}{1600}}} = -11.32$$

Now $\quad |z| > |z_\alpha|$

$\therefore$ The difference between $\bar{x}_1$ and $\bar{x}_2$ (or μ_1 and μ_2) is significant at 1% level.
i.e. H_0 is rejected and H_1 is accepted.
i.e. The Americans are, on the average, taller than the Englishmen.