VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

CAT I Examination – Winter Semester 2023-24

B.Tech – Chemical Engineering

Course: BCHE204L – Transport Phenomena

Time: 90 min **Slot: A2+TA2+TAA2** **Max. Marks: 50**

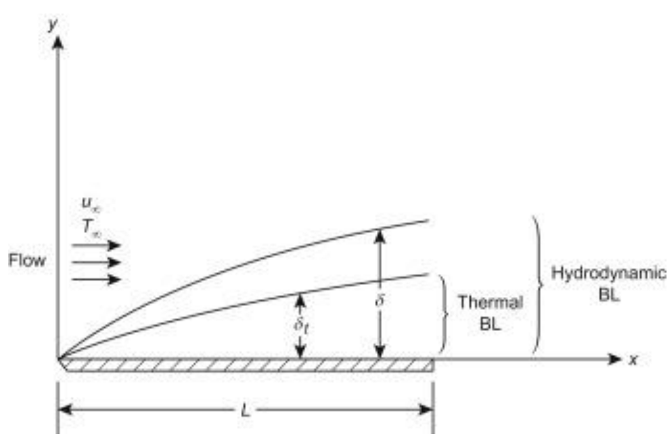
ANSWER KEY

1 (a)	Explain the analogy between momentum and energy transport in terms of relation between their fluxes and gradients, diffusivities and relation between their boundary layer thicknesses. (4M)

Solution

Type of transport	Flux	Driving force	Equation
Momentum	Momentum flux	Velocity gradient	$\tau_{yx} = -\mu \frac{dv}{dy}$
Energy transport	Heat flux	Temperature gradient	$q_x = -k \frac{dT}{dy}$

Prandtl number: $\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$



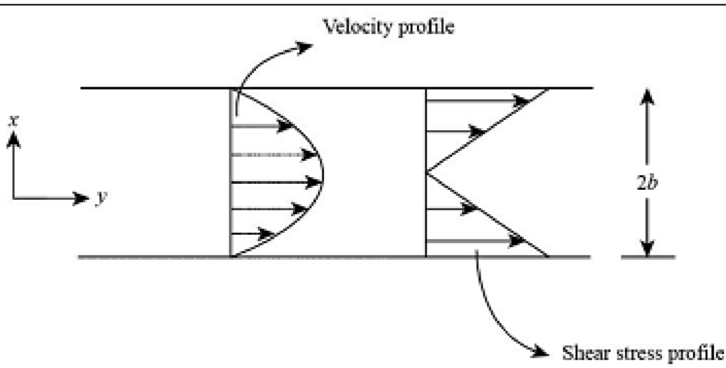
$\delta > \delta_t$ when Pr>1

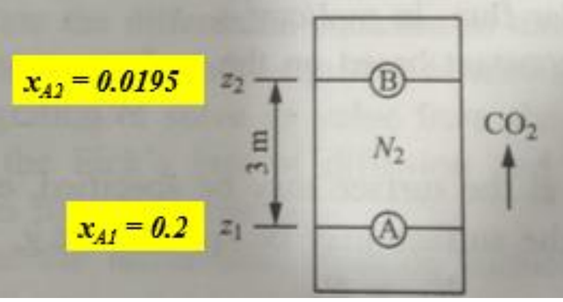
$\delta/\delta_t = (\mathrm{Pr})^{1/3}$

If the Pr = 1, then the hydrodynamic and thermal boundary layers are identical.

If Pr > 1, then $\delta > \delta_t$.

If Pr < 1, then $\delta < \delta_t$.

1 (b)	Determine the heat loss per m^2 from a 40 cm thick furnace wall whose either side surface temperature are maintained at 50 °C and 300 °C. The thermal conductivity of material is given by K = 0.005T – (5×10^{-6}) T^2 (T is in °C). (6M)
Solution	$q_x = -k\frac{dT}{dx}$ $q_x dx = -k dT$ $\int_0^L q_x\, dx = -\int_{50}^{300} k\, dT$ $q_x = -\frac{1}{L}\int_{50}^{300} (0.005T - 5\times10^{-6}T^2)\, dT$ $q_x = -\frac{1}{0.4}(0.005\frac{T^2}{2} - \frac{5\times10^{-6}}{3}T^3\vert_{50}^{300} = 435\ W/m^2$
2 (a)	Draw the shear stress and velocity distribution for the flow of a fluid in a circular conduit under laminar flow condition. (4M)
Solution	
2 (b)	Water at 24 °C is filled between two parallel plates at some distance apart. The lower plate is being pulled at a constant velocity of 0.4 m/s faster relative to the top plate. The viscosity of water at 24 °C is 0.9142×10^{-3} kg/m.s. How far apart (cm) should the two plates be placed so that the shear stress is 0.3 N/m^2? Also, calculate the shear rate. (6M)
Solution	Let Δy be the distance between the two plates. Shear stress, $\tau_{yx} = 0.3$ N/m^2 Applying the Newton's law of viscosity, $\tau_{yx} = -\mu\frac{dv_x}{dy}$ or $\tau_{yx}\int_0^{\Delta y} dy = -\mu\int_{0.4v_x}^{0} dv_x$ or $\tau_{yx} = \mu\frac{v_x}{\Delta y}$

	or $$0.3 = \frac{0.9142 \times 10^{-3} \times 0.4}{\Delta y}$$ $\therefore$ $$\Delta y = 0.9142 \times 10^{-3} \times \frac{0.4}{0.3} = 0.00122 \text{ m}$$ $$= 0.122 \text{ cm}$$ Thus, shear rate, $$\frac{\Delta v_x}{\Delta y} = \frac{0.4}{0.00122} = 327.8 \text{ s}^{-1}$$
3	Carbon dioxide gas is diffusing through nitrogen gas in one direction at atmospheric pressure and temperature of 15 °C. The mole fraction of CO_2 at point A is 0.2 and at point B (3 m away in the direction of diffusion) is 0.0195. The concentration gradient is constant over this distance and diffusivity is 1.5×10^{-5} m^2/s. The gas phase as a whole is stationary, i.e. N_2 is diffusing at the same rate as CO_2, but in opposite direction. (a) What is the molar flux of CO_2 in $kmol/m^2.s$? (5M) (b) What is the molar flux of N_2 in $kmol/m^2.s$? (5M)
Solution	Let A be the CO_2 species and B be the N_2 species. $$\mathcal{D}_{AB} = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$$  Total pressure = 1 atm = 1.0132×10^5 Pa Temperature, T = 15 °C = 273.1+15 = 288.2 K z_2-z_1 = 3 m

Fick's law: $\boxed{J^*_{Az} = -c\mathscr{D}_{AB}\frac{dx_A}{dz}}$ (i)

where c = concentration of the CO_2–N_2 system.

or $c = \frac{P}{RT}$.

Now the molar flux of A becomes

$$J^*_{Az} = -\frac{P}{RT}\mathscr{D}_{AB}\frac{dx_A}{dz} \quad \text{(ii)}$$

where x_A = mole fraction of A.

Integrating Eq. (ii), we have

$$J^*_{Az}\int_{z_1}^{z_2} dz = -\frac{P}{RT}\mathscr{D}_{AB}\int_{x_{A_1}}^{x_{A_2}} dx \quad \text{(iii)}$$

or

$$J^*_{Az}(z_2 - z_1) = \frac{P}{RT}\mathscr{D}_{AB}(x_{A_1} - x_{A_2})$$

or

$$J^*_{Az} = \frac{P}{RT}\mathscr{D}_{AB}\left(\frac{x_{A_1} - x_{A_2}}{z_2 - z_1}\right) \quad \text{(iv)}$$

Substituting the above values in Eq. (iv), we get

$$J^*_{Az} = \frac{1.0132\times10^5\times1.5\times10^{-5}}{8314\times288.1}\left(\frac{0.2-0.0195}{3.0-0}\right)$$

$$= 0.38\times10^{-7}\text{ kg mol of } A/\text{m}^2\cdot\text{s}$$

Ans. (a)

For the component B, i.e. N_2,

mole fraction of B, $x_{B_1} = 1 - 0.2 = 0.8$

mole fraction of B, $x_{B_2} = 1 - 0.0195 = 0.9805$

$\boxed{x_{A1} + x_{B1} = 1.0}$

The mole flux for B, J^*_{Bz}, can be written as

$$J^*_{Bz} = \frac{P\mathscr{D}_{BA}}{RT}\left(\frac{x_{B_1} - x_{B_2}}{z_2 - z_1}\right) \quad \text{(v)}$$

Substituting the values in Eq. (v), we get

$$J^*_{Bz} = \frac{1.0132\times10^5\times1.5\times10^{-5}(0.8-0.9805)}{8314\times288.1\,(3.0\text{-}0)}$$

$$= -0.38\times10^{-7}\text{ kmol of } B/\text{m}^2\cdot\text{s}$$

Ans. (b)

Note: For an ideal gas, $D_{AB} = D_{BA}$

4 The mass flux from a 5 cm diameter naphthalene ball placed in stagnant air at 40ºC and atmospheric pressure is 1.47×10^{-3} mol/m^2. s. Assume the vapour pressure of naphthalene to be 0.15 atm at 40ºC and negligible bulk concentration of naphthalene in air. If air starts blowing across the surface of naphthalene ball at 3 m/s, by what factor will the mass transfer rate increase, all other conditions remaining the same?

	For spheres: $Sh = 2.0 + 0.6\,(Re)^{0.5}\,(Sc)^{0.33}$ Where, Sh is the Sherwood number and Sc is the Schmidt number. The viscosity and density of air are 1.8×10^{-5} kg/m.s and 1.123 kg/m^3, respectively and the gas constant is 82.06 cm^{3}atm/mol.K.
Solution	$Sh = \frac{k_c L}{D_{AB}}$ where L is the characteristic dimension for sphere L = Diameter. $Sc = \frac{\mu}{\rho D_{AB}}$ $R_c = \frac{D v \rho}{\mu}$ $Mass\ flux,\ N_A = K_c \Delta c$ ------------------------------(1) $Sh = 2.0 + 0.6\,(Re)^{0.5}\,(Sc)^{0.33}$ $\frac{k_c D}{D_{AB}} = 2.0 + 0.6\left(\frac{D V \rho}{\mu}\right)^{0.5}\left(\frac{\mu}{\rho D_{AB}}\right)^{0.33}$ ----------------------- (2) also $N = K_G \Delta \bar{p}_A$ $K_G = \frac{k_c}{R T}$ Given: $N = 1.47 * 10^{-3}\ \frac{mol}{m^2.sec} = \frac{K_c}{RT}\Delta \bar{p}_A$ $\frac{k_c}{RT}\left(\frac{0.15}{1} - 0\right) = 1.47 * 10^{-3} * 10^{-4}\ \frac{mol}{cm^2.sec}$ $k_c = \frac{1.47 * 10^{-7}}{0.15} * 82.06 * (273 + 40)$ = 0.0252 cm/sec $k_c = 2.517 * 10^{-4}$ m/sec ------------------------------(3) **Estimation of D_{AB}:** From (2), $\frac{2.517 * 10^{-4} * 5 * 10^{-2}}{D_{AB}} = 2$ (since v = 0) Therefore $D_{AB} = 6.2925 * 10^{-6}$ m^2/sec. $\frac{k_c * 5 * 10^{-2}}{6.2925 * 10^{-6}} = 2 + 0.6\left(\frac{5 * 10^{-2} * 3 * 1.123}{1.8 * 10^{-5}}\right)^{0.5}\left(\frac{1.8 * 10^{-5}}{1.123 * 6.2925 * 10^{-6}}\right)^{0.33}$ $7946\ k_c = 2 + 0.6 * (96.74) * (1.361)$ $k_c = 0.0102$ m/sec. -- (4)

	$\frac{(4)}{(3)} \Rightarrow \frac{N_{A2}}{N_{A1}} = \frac{0.0102}{2.517 * 10^{-4}} = 40.5$ Therefore, rate of mass transfer increases by 40.5 times the initial conditions.
5	Using Buckingham's π-theorem, show that Sherwood Number (***Sh***) is a function of Grashoff (***Gr***) and Schmidt (***Sc***) number for free convection mass transfer. ***Sh = f (Gr, Sc)*** The dimensionless groups can be formulated using mass transfer coefficient k_c' (LT^{-1}) and other variables such as L, ρ, μ, D_{AB}, g and Δρ where L is the pipe length, Δρ is the density difference, μ is the viscosity of the fluid and D_{AB} is the diffusivity of the fluid.
Solution	$f(k_c', L, D_{AB}, g, \mu, \rho) = 0 \rightarrow (1)$ Total number of variables $n = 6$ Number of fundamental dimension, $m = 3$ $\therefore$ Number of π-terms $= n - m = 6 - 3 = 3$ Hence, equation (1) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0$ Choosing D_{AB}, L, μ as repeating variables $\pi_1 = D_{AB}^{a_1} L^{b_1} \mu^{c_1} k_c'$ $\pi_2 = D_{AB}^{a_2} L^{b_2} \mu^{c_2} \rho$ $\pi_3 = D_{AB}^{a_3} L^{b_3} \mu^{c_3} g \Delta\rho$

First - π - term

$$M^0 L^0 T^0 = (L^2 T^{-1})^{a_1} L^{b_1} (M L^{-1} T^{-1})^{c_1} L^1 T^{-1}$$

Equating power of M

$$= L^{2a_1} T^{-a_1} L^{b_1} M^{c_1} L^{-c_1} T^{-c_1} L^1 T^{-1}$$

$$0 = c_1$$

$$c_1 = 0$$

Equating powers of L

$$0 = 2a_1 + b_1 - c_1 + 1$$

$$2a_1 + b_1 + 1 = 0 \quad [\because c_1 = 0$$

Equating power of T

$$0 = -a_1 - c_1 - 1$$

$$a_1 - 1 = 0$$

$$\boxed{a_1 = -1}$$

$$2a_1 + b_1 + 1 = 0$$

$$-2 + b_1 + 1 = 0$$

$$\boxed{b_1 = 1}$$

Sub a_1, b_1 & c_1 in π_1 - term.

$$\pi_1 = D_{AB}^{a_1} L^{b_1} \mu^{c_1} k_c^1$$

$$= D_{AB}^{-1} L^1 \mu^0 k_c^1$$

$$\pi_1 = \frac{k_c^1 L}{D_{AB}}$$

Second π- term

$$\pi_2 = D_{AB}^{a_2} \; b^{2} \; \mu^{c_2} \; \rho$$

$$M^0 L^0 T^0 = [L^2 T^{-1}]^{a_2} [L]^{b_2} [M L^{-1} T^{-1}]^{c_2} \; M L^{-3}$$

$$= L^{2a_2} T^{-a_2} L^{b_2} M^{c_2} L^{-c_2} T^{-c_2} M L^{-3}$$

powers of M

$$0 = c_2 + 1$$

$$\boxed{c_2 = -1}$$

power of L

$$0 = 2a_2 + b_2 - c_2 - 3$$

$$2a_2 + b_2 - c_2 - 3 = 0$$

$$2a_2 + b_2 + 1 - 3 = 0 \qquad [\because c_2 = -1$$

$$2a_2 + b_2 - 2 = 0$$

Power of T

$$0 = -a_2 - c_2$$

$$a_2 - c_2 = 0$$

$$\therefore a_2 = -c_2 \Rightarrow \boxed{a_2 = 1}$$

$$2a_2 + b_2 - 2 = 0$$

$$2 + b_2 - 2 = 0$$

$$\boxed{b_2 = 0}$$

$$\pi_2 = D_{AB}^{a_2} \; L^{b_2} \; \mu^{c_2} \; \rho$$

$$= D_{AB}^{1} \; L^{0} \; \mu^{-1} \; \rho$$

$$= \frac{\rho \, D_{AB}}{\mu}$$

Third - π term

$$\pi_3 = D_{AB}^{a_3} L^{b_3} \mu^{c_3} g \Delta\rho$$

$$M^0 L^0 T^0 = [L^2T^{-1}]^{a_3} [L]^{b_3} [ML^{-1}T^{-1}]^{c_3} ML^{-2}T^{-2}$$

$$= L^{2a_3} T^{-a_3} L^{b_3} M^{c_3} L^{-c_3} T^{-c_3} M^1 L^{-2} T^{-2}$$

Power of M

$$0 = c_3 + 1$$

$$\boxed{c_3 = -1}$$

power of L

$$0 = 2a_3 + b_3 - c_3 - 2$$

$$2a_3 + b_3 + 1 - 2 = 0$$

$$2a_3 + b_3 - 1 = 0$$

Power of T

$$0 = -a_3 - c_3 - 2$$

$$-a_3 - c_3 - 2 = 0$$

$$-a_3 + 1 - 2 = 0$$

$$\boxed{a_3 = -1}$$

$$2(-1) + b_3 - 1 = 0$$

$$b_3 - 3 = 0$$

$$\boxed{b_3 = 3}$$

Sub. a_3, b_3 & c_3 in π_3 term.

$$\pi_3 = D_{AB}^{-1} L^3 \mu^{-1} g \Delta\rho$$

$$\pi_3 = \frac{L^3 g \Delta\rho}{\mu D_{AB}}.$$

As all the π-terms are dimensionless, the following manipulations are possible.

$\pi_1' = \pi_1 \times \pi_2$

$\pi_2' = \pi_2 \times \pi_3$

$\pi_3' = \pi_3 \times \pi_1$

$$\pi_2' = \pi_2 \times \pi_3 = \frac{L^3 g \Delta\rho}{\mu \cdot \cancel{D_{AB}}} \times \frac{\rho \cancel{D_{AB}}}{\mu} = \frac{g L^3 \rho \Delta\rho}{\mu^2}$$

Combining all the π terms;

$$\frac{K_c' L}{D_{AB}} = f\left[\frac{g L^3 \rho \Delta\rho}{\mu^2}, \frac{\mu}{\rho D_{AB}}\right]$$

Sh = f (Gr, Sc)