VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

Fall Semester 2023-2024

Continuous Assessment Test – I

Programme Name & Branch: B.Tech

Slot: B1+TB1

Course Name & code: Calculus -BMAT101L

Exam Duration: 90 Min. **Maximum Marks: 50**

Q.No.	Question	Max Marks	CO	BL
1.	For the function $f(x)=x^4-4x^3+10$, (i) Identify where the extrema of f occurs. (ii) Find the intervals on which f is increasing and the interval on which f is decreasing. (iii) Find where graph of f is concave up and where it is concave down. (iv) Sketch the general shape of the graph for f.	10	CO1	BL1
2.	Sate Lagrange's mean value theorem and also verify it for $f(x)=x^3-x^2-5x+3$ in [0 4].	10	CO1	BL2
3.	Find the volume of the solid of revolution of the region enclosed by the parabola $y=x^2+1$ and the line $x+y=3$ about the $x-$axis.	10	CO2	BL3
4.	(i) Verify the continuity of the function at the origin, where $f(x,y)=\begin{cases}\frac{2xy}{x^2+y^2}, & (x,y)=(0,0)\\ 0, & (x,y)\neq(0,0)\end{cases}$ (ii) Find first and second partial derivatives of $f(x,y)=x^3+y^3-3axy$.	10	CO2	BL1
5.	If $v=f(2x-3y,\,3y-4z,\,4z-2x)$, find $6\frac{\partial v}{\partial x}+4\frac{\partial v}{\partial y}+3\frac{\partial v}{\partial z}$.	10	CO2	BL2

Calculus - BMAT101L - CAT-I - Key (B_1+TB_1 Slot)

1. Critical points $x = 0, 3$ — 10m

(i) Using first derivative test, there is no extremum at $x=0$ and a local minimum at $x=3$

(ii) f is decreasing on $(-\infty, 0]$ and $[0, 3]$ and increasing on $[3, \infty)$

(iii) $f''(x) = 12x^2 - 24x = 12x(x-2)$ is zero at $x=0$ and $x=2$.

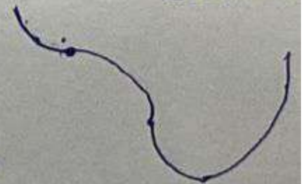
Interval	$x<0$	$0<x<2$	$2<x$
Sign of f''	+	−	+
Behavior of f	Concave up	Concave down	Concave up

$\therefore$ f is concave up on $(-\infty, 0)$ and $(2, \infty)$, and concave down on $(0, 2)$

(iv)

$x<0$	$0<x<2$	$2<x<3$	$3<x$
dec	dec	dec	inc
concave up	conc down	conc up	conc up

Shape of the curve



2. (i) $f(x)$ is continuous in $[0, 4]$ and (ii) derivable in $(0, 4)$ → (10m)

(iii) By L.M.V.T, $\exists$ a point c in $(0,4)$ $\ni$ $f'(c) = \frac{f(4) - f(0)}{4-0}$

i.e., $3c^2 - 2c - 5 = 7 \Rightarrow c = \frac{1 \pm \sqrt{37}}{3}$

We see that $\frac{1+\sqrt{37}}{3} \in (0, 4)$ & thus M.V.T is verified.

3. The two curves intersects in the points, where $x=-2$ & $x=1$

Also, $x^2+1 \leq 3-x$ for all $-2 \leq x \leq 1$. (10 M)

$$\therefore V = \int_{-2}^{1} \pi\left[(3-x)^2 - (x^2+1)^2\right] dx = \frac{117\pi}{5}$$

4(ii) Along the line $y = mx$, — 5m

$$\lim_{\substack{x\to 0\\ y\to 0}} f(x,y) = \lim_{x\to 0} \frac{2mx^2}{x^2+m^2x^2} = \frac{2m}{1+m^2}$$

which is different for the different m selected.

$\therefore \lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$ does not exist.

Consider $\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{2x(0)}{x^2+0} = 0$

$\lim_{y\to 0} f(y,0) = 0$.

$\therefore f(x,y)$ is not continuous at $(0,0)$.

(ii) $f_x = 3x^2 - 3ay$, $f_y = 3y^2 - 3ax$, $f_{xx} = 6x$, $f_{yy} = 6y$. — 5m

Let $r = 2x-3y$, $s = 3y-4z$, $t = 4z-2x$.

(5) $V_x = \frac{\partial V}{\partial x} = \frac{\partial V}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial V}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial V}{\partial t} \cdot \frac{\partial t}{\partial x}$ — 10m

$V_x = 2(V_r - V_t)$

$V_y = \frac{\partial V}{\partial y} = \frac{\partial V}{\partial r} \cdot \frac{\partial r}{\partial y} + \frac{\partial V}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial V}{\partial t} \frac{\partial t}{\partial y}$

$= 3(V_s - V_r)$

$V_z = 4(V_t - V_s)$

$\therefore 6V_x + 4V_y + 3V_z = 0$