Fall Semester 2023-2024

School of Advanced Sciences

Continuous Assessment Test -I (September 2023)

Course: BMAT 101L – Calculus **Slot: B2+TB2**

Max. Time: 90 minutes **Max. Marks:** 50

Answer all the questions (5x10M=50M)

Easy **10 marks** **CO1** **BT2**

1. Verify Rolle 's Theorem for the function $f(x)=e^x(\sin x-\cos x)$ in $\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$.

Moderate **10 marks** **CO1** **BT1**

2. Let $f(x)=x\left(\frac{x}{2}-5\right)^4$. Find (i) the critical points of f (ii) the intervals on which is f increasing and f is decreasing (iii) the local minima and maxima of f (iv) Determine the intervals on which f is concave up and concave down (v) the point of inflection.

Tough **10 marks** **CO1** **BT3**

3. Find the volume of the solid generated by revolving the region bounded by the parabola $y=x^2$ and the line $y=1$ about (a) the line $y=1$ (b) the line $y=2$

Easy **(5+5=10) marks** **CO2** **BT1**

4. (a) Find the limit of $\lim\limits_{\substack{(x,y)\to(2,2)\\(x+y)\neq 4}}\left[\frac{x+y-4}{\sqrt{x+y}-2}\right]$

(b) Test the continuity at $(0,0)$ of the function

$$f(x,y)=\begin{cases}\dfrac{x^2}{\sqrt{x^2+y^2}}, & for\,(x\neq 0, y\neq 0)\\ 3, & \text{for } (x=0, y=0)\end{cases}.$$

Moderate **10marks** **CO2** **BT2**

5. Find $\frac{du}{dx}$, if $u=\sin\left(x^2+y^2\right)$, where $a^2x^2+b^2y^2=c^2$.

Fall Semester-2023

BMAT-101L- B2+TB2-key

① Statement: If a function $f(x)$ is (i) Continuous in the $[a,b]$ (ii) differentiable in the (a,b) (iii) $f(a)=f(b)$ then there exists atleast one point $c\in(a,b)$ such that $f'(c)=0$

$$f(x)=e^{x}(\sin x-\cos x) \text{ in } \left[\frac{\pi}{4},\frac{5\pi}{4}\right]$$

$$f'(x)=2e^{x}\sin x$$

$$f\left(\frac{\pi}{4}\right)=f\left(\frac{5\pi}{4}\right)=0$$

By Rolle's Theorem $f'(c)=0$

$$2e^{c}\sin c=0 \Rightarrow \sin c=0$$

$$c=0,\pi,2\pi,\ldots$$

$c=\pi$ lies in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

② $f(x)=x\left(\frac{x}{2}-5\right)^{4} \Rightarrow f'(x)=\left(\frac{x}{2}-5\right)^{\cancel{4}\,3}\left(\frac{5x}{2}-5\right)$

$$f''(x)=5\left(\frac{x}{2}-5\right)^{2}(x-4)$$

The curve is rising on $(-\infty,2)$ and $(10,\infty)$ and falling on $(2,10)$.

Local maximum at $x=2$

Local minimum at $x=10$

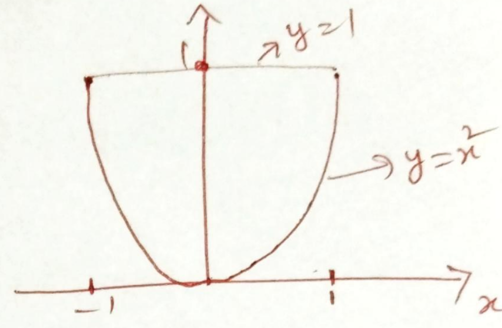
Concave down on $(-\infty,4)$

Concave up on $(4,\infty)$

At $x=4$ there is a point of inflection

(3) Given parabola $y = x^2$

$r(x) = 0$ and $R(x) = 1 - x^2$



(a) $V = \int_{-1}^{1} \pi \left[(R(x))^2 - (r(x))^2 \right] dx$

$= \pi \int_{-1}^{1} (1-x^2)^2 dx = \frac{16}{15} \pi$

(b) $r(x) = 1$ and $R(x) = 2 - x^2$

$V = \pi \int_{-1}^{1} \left[(2-x^2)^2 - 1 \right] dx = \frac{56}{15} \pi$

4 (a) $\lim_{(x,y) \to (2,2)} \frac{x+y-4}{\sqrt{x+y}-2} = \lim_{\substack{x \to 2 \\ y \to 2}} \frac{(\sqrt{x+y}+2)\cancel{(\sqrt{x+y}-2)}}{\cancel{\sqrt{x+y}-2}}$

$= \lim_{\substack{x \to 2 \\ y \to 2}} \sqrt{x+y} + 2 = 4.$

(b) $\lim_{\substack{x \to 0 \\ y \to 0}} \frac{x^2}{\sqrt{x^2+y^2}} = 0$ and $\lim_{\substack{y \to 0 \\ x \to 0}} \frac{x^2}{\sqrt{x^2+y^2}} = 0$

Along $y = mx$: $\lim_{\substack{y \to mx \\ x \to 0}} \frac{x^2}{\sqrt{x^2+y^2}} = 0$

Along $y = mx^2$: $\lim_{\substack{y \to mx^2 \\ x \to 0}} \frac{x^2}{\sqrt{x^2+y^2}} = 0$

But given $f(0,0) = 3$

$f(x,y)$ is not continuous at the origin.

(5) Given that $u = \sin(x^2+y^2)$

We know that $\frac{du}{dx} = \frac{\partial u}{\partial x}\frac{dx}{dx} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dx} \rightarrow (1)$

$u = \sin(x^2+y^2)$

$\frac{\partial u}{\partial x} = 2x\cos(x^2+y^2)$ and $\frac{\partial u}{\partial y} = 2y\cos(x^2+y^2)$

Given that $a^2x^2+b^2y^2 = c^2 \rightarrow (2)$

Differentiating $a^2x^2+b^2y^2 = c^2$ w.r.t x

$2a^2x + 2b^2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{a^2x}{b^2y}$

$(1) \Rightarrow \frac{du}{dx} = 2x\cos(x^2+y^2) + 2y\cos(x^2+y^2)\cdot\left(-\frac{a^2x}{b^2y}\right)$

$= 2\cos[x^2+y^2]\left\{x - \frac{a^2x}{b^2y}\right\}$

$\frac{du}{dx} = 2\frac{(b^2-a^2)x}{b^2}\cos(x^2+y^2)$