SCHOOL OF ADVANCED SCIENCES

Fall Semester 2023-24

Continuous Assessment Test – II

Programme Name & Branch: B.Tech

Slot: G2+TG2

Course Name & Code: Calculus- BMAT101L

Exam Duration: 90 Min — Maximum Marks=50

Q. No.	Question	Marks	CO	BL
1.	Check whether the functions $f(x,y)$ and g(x,y) are functionally dependent. If so, find the relation between them, where $$f(x,y)=\frac{x+y}{1-xy},\;\; g(x,y)=\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}$$	10	**CO2**	BL2
2.	Let $f(x,y)=x\sqrt{y}$. Use a 2nd order Taylor's approximation to approximate $f(5.9,4.1)$.	10	**CO2**	BL2
3.	Using Lagrange's multipliers method find the largest product of the positive numbers x, y and z such that $x+y+z^2=16$.	10	**CO3**	BL2
4.	Sketch the region of integration, and evaluate $$I=\int_0^6\left(\int_{x/3}^{2} x\sqrt{1+y^3}\,dy\right)dx.$$	10	**CO3**	BL3
5.	Calculate the volume under the surface $z=3+x^2-2y$ over the region D defined by $0\le x\le 1$ and $-x\le y\le x$ by converting into polar coordinates.	10	**CO3**	BL3

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CAT-2 Key BMAT101L - Calculus - A2+TA2

Q1 $$\frac{\partial(f,g)}{\partial(x,y)} = \begin{vmatrix} \frac{1+y^2}{(1-xy)^2} & \frac{1+x^2}{(1-xy)^2} \\ \frac{(1-xy)^2-(x+y)^2}{(1+x^2)^2(1+y^2)} & \frac{(1-xy)^2-(x+y)^2}{(1+x^2)(1+y^2)^2} \end{vmatrix} = 0$$

Here f and g are functionally dependent.

Now, $f = \frac{x+y}{1-xy}$, $\Rightarrow x = \frac{f-y}{1+fy}$

Substitute in $g = \frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}$

$$= \frac{f(1-xy)^2}{(1+x^2)(1+y^2)} \quad \text{— (1)}$$

Now, $1+x^2 = 1+\frac{(f-y)^2}{(1+fy)^2} = \frac{(1+f^2)(1+y^2)}{(1+fy)^2}$ — (2)

From (1) and (2)

$$g = \frac{f}{1+f^2}\left\{\frac{(1-xy)(1+fy)}{1+y^2}\right\}^2 \qquad \Big| \ (1-xy)(1+fy) = 1+y^2$$

$$= \frac{f}{1+f^2}$$

Q2 Since (6,4) is the closest point to (5.9, 4.1) therefore we will do taylor series expansion at (6,4)

$f(x,y) = x\sqrt{y}$ $\quad$ $f(6,4) = 12$

$f_x = \sqrt{y}$ $\quad$ $f_x = 2$

$f_y = \frac{x}{2\sqrt{y}}$ $\quad$ $f_y = 3/2$

$f_{xx} = 0$ $\quad$ $f_{xx} = 0$

$f_{xy} = \frac{1}{2\sqrt{y}}$ $\quad$ $f_{xy} = \frac{1}{4}$

$f_{yy} = \frac{x}{4y^{3/2}}$ $\quad$ $f_{yy} = -\frac{3}{16}$

$$f(x,y) = 12 + 2(x-6) + \frac{3}{2}(y-4) + \frac{1}{2}\left\{ -\frac{3}{16}(y-4)^2 + 2(x-6)(y-4)\frac{1}{4} + 0 \right\}$$

$f(5.9, 4.1) \approx 11.95$

Q3 $f(x,y,z) = xyz$; $g(x,y,z) = x+y+z^2-16$

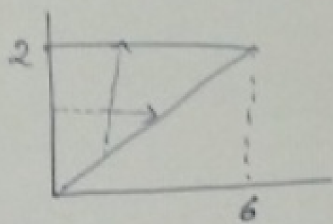
$$\nabla f = \lambda \nabla g \Rightarrow \left.\begin{array}{l} yz = \lambda \\ xz = \lambda \\ xy = 2\lambda z \end{array}\right\} \Rightarrow x = y = 2z^2$$

Substituting in $x+y+z^2=16$ we get .

$x = \frac{32}{5}$, $y = \frac{32}{5}$, $z = \frac{4}{\sqrt{5}}$,

Largest product = 73.2714.

Q4 We need to swap the order of integration



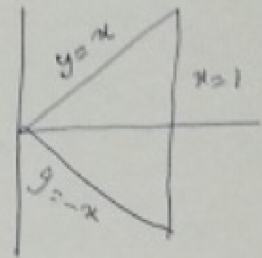
Horizontal strip

$$0 \le x \le 3y, \quad 0 \le y \le 2$$

$$I = \int_0^2 \int_0^{3y} x\sqrt{1+y^3}\,dx\,dy = \frac{9}{2}\int_0^2 y^2\sqrt{1+y^3}\,dy$$

$$= 26 \qquad \Big| \; u = 1+y^3$$

Q5 Let $x = r\cos\theta$, $y = r\sin\theta$



$$dx\,dy = r\,d\theta\,dr$$

$$0 \le r \le \sec\theta$$

$$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

$$V = \int_{-\pi/4}^{\pi/4} \int_0^{\sec\theta} \left(3 + r^2\cos^2\theta - 2r\sin\theta\right) r\,dr\,d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \left(\frac{7}{4}\sec^2\theta - \frac{2}{3}\tan\theta\sec^2\theta\right) d\theta$$

$$= \left| \frac{7}{4}\tan\theta - \frac{\tan^2\theta}{3} \right|_{-\pi/4}^{\pi/4} = \frac{7}{2}$$