



# VIT

Vellore Institute of Technology  
(Approved by AICTE, Ministry of Education, Government of India)

## Final Assessment Test - Jan / Feb 2023

Course: **BMAT101L** - Calculus

Class NBR(s): 5425/5477/5667/5701/6420

Time: Three Hours

Slot: A1+TA1

Max. Marks: 100

**KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE**

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. For the function  $f(x) = 2x^3 - 14x^2 + 22x - 6$ ,
- Find the intervals on which  $f$  is increasing and on which  $f$  is decreasing.
  - Find the local maxima and local minima using first derivative test.
  - Identify the intervals where the function is concave up and concave down, hence find the points of inflection.

2. a) Find the area of the region enclosed by the curve  $y = x^2 - 2$  and the line  $y = 2$ . [5]

- b) Find the volume of the solid when the region enclosed by  $y = \sqrt{x+1}$ ,  $y = 2\sqrt{x}$ ,  $x = \frac{1}{3}$  and  $x = 1$  is revolved about x-axis. [5]

3. a) Given  $u = \frac{x^2 - y^2}{x^2 + y^2}$  and  $v = \frac{2xy}{x^2 + y^2}$ . Test whether  $u$  and  $v$  are [5]

functionally dependent. If so, state the relationship between them.

- b) Investigate the continuity of the function [5]

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

4. Expand  $f(x, y) = xe^{-y} + \sin x \cos y$  as a Taylor's series in powers of  $x$  and  $y$  up to second degree terms.

5. Find the shortest and the longest distances from the point  $(1, 2, -1)$  to the sphere  $x^2 + y^2 + z^2 = 24$ .

6. Change the order of integration and evaluate  $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4 - a^2x^2}} dy dx$ .

7. Using cylindrical coordinates find the volume intercepted between the paraboloid  $x^2 + y^2 = 2az$  and the cylinder  $x^2 + y^2 - 2ax = 0$

8. a) Evaluate  $\int_0^1 \frac{30x^2}{32\sqrt{1-x}} dx$  using Beta and Gamma functions. [5]

- b) Evaluate  $\int_0^2 (8 - x^3)^{-\frac{1}{3}} dx$  using Beta and Gamma functions. [5]

9. a) If  $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$ , find  $\text{div}\vec{F}$ . [5]

b) If  $\vec{A} = 3xz^2\vec{i} - yz\vec{j} + (x + 2z)\vec{k}$  find  $\text{curl}(\text{curl}\vec{A})$ . [5]

10. A fluid motion is given by

$\vec{F} = (e^x \cos y + yz)\vec{i} + (xz - e^x \sin y)\vec{j} + (xy + z)\vec{k}$ . Show that the motion is irrotational and hence find its scalar potential  $\phi$ .

11. Using Stokes' theorem evaluate  $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$

where C is the boundary of the triangle with vertices (2,0,0); (0,3,0) and (0,0,6).

12. If  $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$  using Gauss divergence theorem, evaluate

$\iint_S \vec{F} \cdot \hat{n} ds$  where S is the closed surface bounded by the planes

$z = 0, z = 1$  and the cylinder  $x^2 + y^2 = 4$ .

