VIT
Vellore Institute of Technology

Final Assessment Test– Jan/Feb 2023

Course: BMAT101L - Calculus

Class NBR(s): 5421 / 5460 / 5474 / 5542 / 6209 Slot: A2+TA2

Time: Three Hours Max. Marks: 1

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRAC

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Find the area of the region under the graph of $f(x)=x\sqrt{4-x^2}$ between the ordinates $x=-2$ and $x=2$. Further, use washer's method to obtain the volume of the solid generated by revolving the curve $y=f(x)$ between the limits $x=-2$ and $x=2$. [10]

2. a) State Mean value theorem and verify that the Mean value theorem applies for the function $f(x)=x^3+3x^2-24x$ on the interval [1,4]. [5+5]

   b) Find the absolute maximum and minimum values of
   $f(x) = x^3 -3x^2+1, -1/2 \le x \le 4$

3. If $u = x + 2y + z$, $v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$, show that they are not independent. Find the relation between u, v and w. [10]

4. Let $f(x,y)=\sin 2x \cos 3y$. Then find all the partial derivatives of upto third order at the origin, and then obtain a cubic approximation of f near the origin specify? [10]

5. The temperature at a point (x,y) on a metal plate is $T(x,y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant? [10]

6. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$ and hence evaluate it. [10]

7. Evaluate $\iiint (x+y+z)\, dxdydz$ over the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$. [10]

8. a) Prove that $\Gamma(1/2) = \sqrt{\pi}$ and hence find $\int_0^\infty e^{x^2}\, dx$ [5]

   b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta}\; d\theta$ [5]

9. a) Find the direction in which temperature changes most rapidly with distance from the points (1, 1, 1) and determine the maximum rate of change if the temperature at any point is given by $f(x, y, z) = xy + yz + zx$. [5+5]

   b) Determine the constant b such that $\bar{A} = (x + 3y)\,\hat{\imath} + (y - 2z)\,\hat{\jmath} + (x + bz)\,\hat{k}$ is solenoidal

10. Verify that $\vec{F} = (6xy+z^3)\vec{\imath} + (3x^2-z)\vec{\jmath} + (3xz^2-y)\vec{k}$ is a Conservative field? Also find it's scalar potential function [10]

11. Verify Green's theorem for $\oint_C [(x^2-2xy)\,dx + (x^2y+3)dy]$ along the curves bounded by $y^2 = 8x$ and $x = 2$ [10]

12. Use Gauss' divergence theorem to compute $\iint F.nds$ over the surface of the sphere $x^2+y^2+z^2=a^2$, where $\bar{F} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$ [10]

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