Final Assessment Test – Jan/Feb 2023

VIT
Vellore Institute of Technology

Course: BMAT101L - Calculus

Class NBR(s): 5008/ 5012/ 5015/ 5017/5022/ 5027/ 5029/5031/5034/5035/5037/5041/5043/5046/5049/ 5051/5418/ 5424/5483/5490/6423/6443

Slot: B1+TB1

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. a) Use the first derivative test to find the location of all local extrema for. $f(x) = x^3-3x^2-9x-1$. Sketch the graph to confirm your results. Also find the region where the function increasing and decreasing. [5]

   b) Find the area of the region bounded by the parabolas $y=6x-x^2$ and $y=x^2-2x$. [5]

2. a) Find a point on the curve $y = \sin x + \cos x - 1$, $x \in [0, \pi/2]$, where the tangent is parallel to the x axis. [5]

   b) A solid is formed by rotating the triangle with vertices (0, 0), (2, 0) and (1, 1) about x-axis. Find the resulting volume. [5]

3. a) If $u = x\log(xy)$ where $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$. [5]

   b) Prove that if $u = \frac{x}{y}, and\ v = \frac{x+y}{x-y}$ are functionally dependent and find the relation between them. [5]

4. Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem up to third degree terms. [10]

5. Find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = 1$ [10]

6. Evaluate the volume of the region enclosed by the surfaces $z=x^2+3y^2$ and $z=8-x^2-y^2$ by using the idea of triple integral. [10]

7. Change the order of integration and evaluate $\int_0^a \int_0^y \frac{dxdy}{\sqrt{(a^2+x^2)(a-y)(y-x)}}$. [10]

8. Evaluate $\iiint \frac{dxdydz}{(x+y+z+1)^3}$, the integral being taken throughout the volume bounded by planes x=0, y=0, z=0, x+y+z=1 by using Beta-Gamma Functions. [10]

9. Find the directional derivative of $\nabla.(\nabla\phi)$ at the point (1, -2, 1) in the direction of the normal to the surface $xy^2z = 3x + z^2$, Where $\phi = 2x^3y^2z^4$. [10]

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10. Show $F = 2xy^3z^4\, i + 3x^2y^2z^4\, j + 4x^2y^3z^3\, k$ is conservative over its natural domain and find a potential function for it. [10]

11. Verify Green's theorem for $\oint_c (xy^2\,dx + x^2y\,dy)$ where 'C' in the circle $x^2 + y^2 = 4$. [10]

12. Verify the divergence theorem for $\bar{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z=0$, and z = 3. [10]

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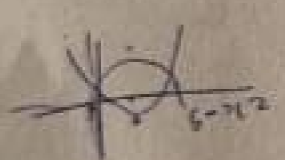

A -2

1-2
(3-4)
5&6 F-(
(7-8)
(9-10)
(11)-12