VIT
Vellore Institute of Technology

Final Assessment Test – November/December 2023

Course: IMAT101L - Calculus

Time: Three Hours   Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. (i) If Rolle's theorem holds for $f(x) = x^3 + bx^2 + cx,\ -2 \le x \le 2$ at the point $x = 1$. Find the values of b and c.

   (ii) Find the local extreme values of the given function $f(x) = x^2 e^{-x}$.

2. Find the intervals in which the following curve is concave upward or concave downward. Also find the points of inflection.
$$f(x) = 3x^4 + 4x^3 - 6x^2 + 12x + 12$$

3. If $u = u(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

4. If $u = xy + yz + zx, v = x^2 + y^2 + z^2$ and $w = x + y + z$, determine whether there is a functional relationship between u, v, w and if so, find it.

5. Expand $x^2y + 3y - 2$ in powers of $x - 1$ and $y + 2$ using Taylor's Theorem as far as the terms of third degree.

6. Show that the following function is maximum at $(-7,-7)$ and minimum at $(3,3)$, also find the absolute maximum and minimum values.
$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$$

7. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dydx$ and hence evaluate it.

8. Evaluate $\iint_R (x + y)^2\, dxdy$, where R is the parallelogram in the xy −plane with vertices $(1,0), (3,1), (2,2), (0,1)$ using the transformation $u = x + y$ and $v = x - 2y$.

9. (i) Evaluate $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}} dx$.

   (ii) Show that $\int_0^{\frac{\pi}{2}} \sin^p \theta\, \cos^q \theta\; d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$.

10. A fluid motion is given by $\vec{v} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$. Show that the motion is irrotational and hence find the velocity potential.

11. State and verify Green's theorem in the plane for $\int_C (3x^2 - 8y^2)dx + (4x - 6xy)dy$, where C is the boundary of the region bounded by $x \ge 0,\ y \ge 0$ and $2x - 3y = 6$.

12. Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - (xz)\vec{k}$ over the surface of a cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the XOY plane.

⇔⇔⇔