



VIT

Vellore Institute of Technology

Final Assessment Test - November/December 2023

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2002 / 2003 / 2004 / 2005 / 2006 / 2016 /

2017 / 2019 / 2020 / 2021 / 2022 / 2023 / 2027 / 2028 / Slot: A1+TA1+TAA1  
5005

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

- Let  $f = u + iv$  where  $u = x^2 - y^2 + 2xy$  and  $v = x^2 - y^2 - 2xy$ . Show that  $u$  and  $v$  are harmonic and determine whether  $f$  is an analytic function in the complex plane.
- Let  $\phi$  be the potential function given by  $\phi(x, y) = e^{2x}(x \cos(2y) - y \sin(2y))$ . Find the stream function  $\psi$  such that  $f = \phi + i\psi$  is analytic.
- Let  $f(z) = \sqrt{2} e^{i\pi/4} z + (1 - 2i)$ .
  - Find the coefficient of magnification and angle of rotation at the point  $2 + 3i$ .
  - Find the image of the rectangular region  $0 \leq x \leq 2$ ,  $0 \leq y \leq 2$  under the transformation of  $f$ .
- Find the bilinear transformation which maps the points  $z = 1, i, -1$  into  $w = i, 0, -i$ . Also, find the image of  $|z| < 1$ .
- Expand  $f(z) = \frac{z+3}{z(z^2-z-2)}$  the Laurent's series for the region
  - $|z| < 1$
  - $1 < |z| < 2$
  - $|z| > 2$ .
- Let  $f(z) = \frac{z+2}{(z+1)^2(z-2)}$ .
  - Find the residue of  $f$  about each singularity.
  - Evaluate  $\oint_C f(z) dz$  over the curve  $C: |z - i| = 2$  using Cauchy's residue theorem.
- Let  $A = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{bmatrix}$ . Find the basis of  $R(A)$ ,  $C(A)$  and  $N(A)$ . Also determine, whether the vector  $(1, 1, 1)$  is in  $R(A)$ .
- Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a transformation defined by  
 $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$ . Show that  $T$  is invertible and find  $T^{-1}(u, v, w)$ .
- Find the matrix of change of basis  $A$  from the basis  $\alpha = \{x^2 + x + 1, x^2 + 1, x - 1\}$  and  $\beta = \{2x^2 + 3x + 1, 2x^2 + 2x + 1, -x^2 - 2\}$  for  $P_2(x)$ . Use this result and find the change of basis from  $\beta$  to  $\alpha$ .

10. Find an orthonormal basis for the subspace  $U$  of  $R^4$  spanned by the vectors  $v_1 = (1,1,1,1)$ ,  $v_2 = (1,2,4,5)$ ,  $v_3 = (1, -3, -4, -2)$  using Gram Schmidt orthogonalization process.

11. Find Characteristic equation of  $A$  and  $A^{-1}$  using Cayley Hamilton theorem, when

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}.$$

12. Using Gauss Jordan method, solve the following system of equations

$$x + y - z - w = 2,$$

$$x - y - z + w = 4,$$

$$2x + y - z + w = 1.$$

⇔⇔⇔