



SCHOOL OF ADVANCED SCIENCES

FALL Semester 2023-2024
Continuous Assessment Test - I

Programme Name & Branch : B.Tech.,

Slot: CI+TC1+TCC1

Course Name & code: Complex Variables and Linear Algebra & BMAT201L

Exam Duration: 90 Min.

Maximum Marks: 50

Answer all questions (5X10=50)

1. Show, by considering the function $f(z)$ defined by $f(z) = \frac{2xy(x+iy)}{x^2+y^2}$ for $z \neq 0$ and $f(0) = 0$, that the C-R equations are not the sufficient conditions for a function to be analytic.

2. Show that $v(x, y) = x^3 - 3x^2y + 2x + 1 + y^3 - 3xy^2$ is a harmonic function and obtain an analytic function for which $v(x, y)$ is the imaginary part. Also find the conjugate harmonic of $v(x, y)$.

3. Find the Bilinear transformation which maps $z_1 = -2i$, $z_2 = i$, $z_3 = \infty$ onto $w_1 = 0$, $w_2 = -3$, $w_3 = \frac{1}{3}$ respectively. Also find

i) the invariant points of the transformation.

ii) the image of $|z| < 1$ in the w-plane.

4. Find the Laurent's series for the function $f(z) = \frac{1}{(z+1)(z+2)^2}$ in the following regions:

i) $|z-1| < 2$

ii) $2 < |z-1| < 3$

5. a) Find the location and nature of singularity of $f(z) = \frac{1-e^z}{z^3}$.

b) Using Cauchy integral formula, evaluate $\int_C \frac{z+1}{z^3-2z^2} dz$, where C is a circle $|z-2-i|=2$.

(4M+6M)
