**VIT**
**Vellore Institute of Technology**
(Deemed to be University under section 3 of UGC Act, 1956)

# School of Advanced Sciences
## Fall Semester 2023-24
## Continue Assessment Test -II

**Programme Name and Branch:** B.Tech. **Slot: A1+TA1+TAA1**
**Course Name:** Complex Variables and Linear Algebra **Course Code:** BMAT201L
**Exam Duration: 90 Min.** **Maximum Marks: 50**

**General instruction(s):** Attempt all questions

| Q. No. | Question | Max Marks | CO | BL |
|---|---|---|---|---|
| **1.** | Evaluate the following real improper integral using Cauchy's Residue theorem: $$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2-4x+5)}\,dx.$$ | **10** | 3 | 5 |
| **2.** | Discuss the type of consistency and hence find the solution set of the following system of equations using Gauss Elimination method: $$x_1+x_2+2x_3+x_4=7$$ $$2x_1+x_2+2x_3-x_4=6$$ $$3x_1+5x_2+10x_3+9x_4=37$$ | **10** | 5 | 2 |
| **3.** | **a)** Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$. Also verify the Cayley-Hamilton Theorem. | **5** | 5 | 5 |
| | **b)** A matrix $B_{2\times 2}$ has Eigen values 2 and 3. The corresponding Eigen vectors are $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$ respectively. Then find the matrix B. | **5** | | |
| **4.** | Show the space $U=\{(k+l+m,\ k-3l,\ l-2m,\ m+k);\ k,l,m\in\mathbb{R}\}$ is forming a subspace of $\mathbb{R}^4$. Also, find basis and dimension of subspace $U$. | **10** | 5 | 3 |
| **5.** | Examine whether the following set of vectors and linearly independent or linearly dependent: **a)** $W_1=\{4t^2-t+2,\ 3t^2-4-t,\ 1-t\}$. **b)** $W_2=\left\{\begin{bmatrix}1&1&0\\0&0&2\end{bmatrix},\begin{bmatrix}2&0&1\\0&-1&0\end{bmatrix},\begin{bmatrix}1&1&0\\0&2&3\end{bmatrix}\right\}$. | **5+5** | 5 | 4 |

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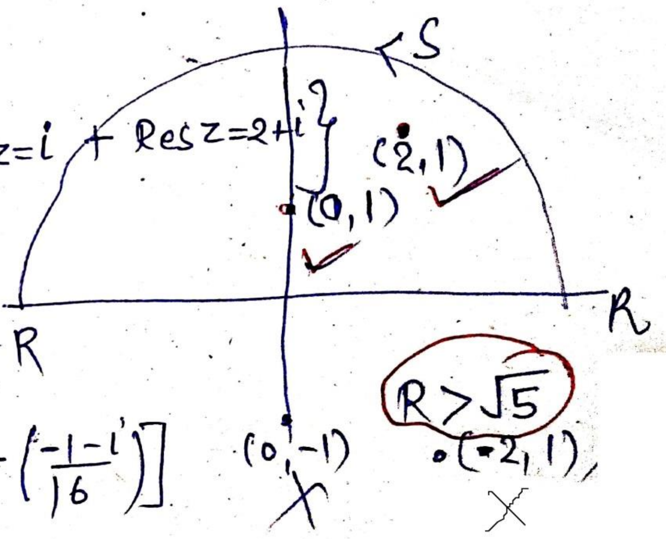
**Solution 1:** let $\int_C \frac{1}{(z^2+1)(z^2+4z+5)}\,dz$

$z = \pm i,\ 2 \pm i$

$\int_C = \int_{-R}^{R} + \int_S = 2\pi i \{\text{Res}\ z=i + \text{Res}\ z=2+i\}$




$R \to \infty \quad \int_S = 0$

$\Rightarrow \int_{-\infty}^{\infty} = 2\pi i\left[\frac{1-i}{16} + \left(\frac{-1-i}{16}\right)\right]$

$= \pi/4$ #

**Solution 2:**

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 7 \\ 2 & 1 & 2 & -1 & 6 \\ 3 & 5 & 10 & 9 & 37 \end{array}\right]$$

$R_3 \to R_3 - 3R_1$, $R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 7 \\ 0 & -1 & -2 & -3 & -8 \\ 0 & 2 & 4 & 6 & 16 \end{array}\right]$$

$R_3 \to R_3 + 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 7 \\ 0 & -1 & -2 & -3 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho = R < n$ ($\infty$ many soln)

$n - \rho = 4 - 2 = 2$ free variables $\quad t_1, t_2 \in \mathbb{R}$

set $z = t_1 \quad w = t_2$

$y + 2z + 3w = 8 \Rightarrow y = 8 - 2t_1 - 3t_2$

$x + y + 2z + 3w = 7 \Rightarrow x = 7 - 2t_1 - 3t_2 - (8 - 2t_1 - 3t_2)$

$x = 2t_2 - 1$

solution set $= \{(2t_2 - 1,\ 8 - 2t_1 - 3t_2,\ t_1,\ t_2);\ t_1, t_2 \in \mathbb{R}\}$

**Solution 3A:**

$\lambda_1 = -2,$ $\quad X_1 = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

$\lambda_2 = -1$ $\quad X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$f(\lambda) = \lambda^2 + 3\lambda + 2 = 0$$

$$A^2 + 3A + 2I = 0$$

$$\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}^2 + 3\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$0 = 0 \quad \#$$

**Solution 3B:**

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$BX_1 = \lambda_1 X_1$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$a + b = 2$

$c + d = 2$

$a + b = 2$

$-\ a + 2b = 3$

$-b = -1$

$b = 1$

$a = 1$

$$B = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix}$$

$$BX_2 = \lambda_2 X_2$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$a + 2b = 3$

$c + 2d = 6$

$c + d = 2$

$c + 2d = 6$

$-d = -4$

$d = 4$

$c = 2 - d = -2$

**Solution 4**

Let $U = (k_1 + l_1 + m_1,\ k_1 - 3l_1,\ l_1 - 2m_1,\ m_1 + k_1)$

$V = (k_2 + l_2 + m_2,\ k_2 - 3l_2,\ l_2 - 2m_2,\ m_2 + k_2)$

Let $\alpha, \beta \in R$.

then
$$\alpha U + \beta V = \begin{Bmatrix} (\alpha k_1 + \beta k_2) + (\alpha l_1 + \beta l_2) + (\alpha m_1 + \beta m_2), \\ (\alpha k_1 + \beta k_2) - 3(\alpha l_1 + \beta l_2), \\ (\alpha l_1 + \beta l_2) - 2(\alpha m_1 + \beta m_2), \\ (\alpha m_1 + \beta m_2) + (\alpha k_1 + \beta k_2) \end{Bmatrix} \in U$$

$\therefore$ U is a subspace of $R^4$ #

$$\begin{aligned} (k+l+m,\ k-3l,\ l-2m,\ m+k) &= (k, k, 0, k) + (l, -3l, l, 0) + (m, 0, -2m, m) \\ &= k(1,1,0,1) + l(1,-3,1,0) + m(1,0,-2,1) \end{aligned}$$

$\therefore$ $S = \{\underset{U_1}{(1,1,0,1)},\ \underset{U_2}{(1,-3,1,0)},\ \underset{U_3}{(1,0,-2,1)}\}$

(1) $L(S) = U$

(2) S is L.I. $\rightarrow$ $\because \left[ \begin{aligned} \alpha_1 U_1 + \alpha_1 U_2 + \alpha_3 U_3 = 0 \\ \alpha_1 = \alpha_2 = \alpha_3 = 0 \end{aligned} \right]$

$\therefore$ S is basis for U #

$\text{Dim}\, U = 3$ #

**Solution 5A:** $\alpha(4t^2 - t + 2) + \beta(3t^2 - t - 4) + \gamma(-t + 1) = 0$

$$(4\alpha + 3\beta)t^2 + (-\alpha - \beta - \gamma)t + (2\alpha - 4\beta + \gamma) = 0 \cdot t^2 + 0 \cdot t + 0$$

$\Rightarrow$

$$4\alpha + 3\beta = 0$$
$$\alpha + \beta + \gamma = 0$$
$$2\alpha - 4\beta + \gamma = 0$$

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 3 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & -4 & 1 & 0 \end{array}\right]$$

Det of $\begin{pmatrix}\text{Coeff} \\ \text{Matrix}\end{pmatrix} = 23 \neq 0 \quad \therefore$ $W_1$ is L.I. #

**Solution 5B:**

$$\alpha\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \beta\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} + \gamma\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} \alpha + 2\beta + \gamma & \alpha + \gamma & \beta \\ 0 & -\beta + 2\gamma & 2\alpha + 3\gamma \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\alpha = \beta = \gamma = 0, \quad \therefore$ $W_2$ is L.I. #