8. Let $T: R^3(R) \to R^3(R)$ be a linear transformation defined by $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$ for all $x, y, z \in R$. Find T^{-1} if T is invertible.

9. Let $T: R^3(R) \to R^2(R)$ be a linear transformation defined by $T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$ for all $x, y, z \in R$. Find the matrix of linear transformation T relative to the base $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $B_2 = \{(1, 3), (1, 5)\}$.

10. Obtain an orthogonal basis for the subspace of $R^4(R)$ spanned by $\alpha_1 = (1, 0, 1, 0)$, $\alpha_2 = (1, 1, 1, 1)$, $\alpha_3 = (-1, 2, 0, 1)$ using Gram Schmidt orthogonal process.

11. Let $A = \begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$. Use Cayley-Hamilton theorem to find the values of a, b and c such that $A^4 = aA^2 + bA + cI$. $a = 129$ $b = 214$ $c = 109$

12. Solve the following system of linear equations using Gauss Jordan method.

$$x + 2y - z - w = 2,$$
$$2x - y + 3z + w = 7,$$
$$x - 4y + 3z - 2w = -3,$$
$$x - y + 4z + w = 8.$$

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VIT
Vellore Institute of Technology

Final Assessment Test – November/December 2023

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2030 / 2031 / 2032 / 2036 / 2037 / 2038 / 2041 / 2042 / 2043 / 2044 / 2046 / 2049 / 2050 / 2051 / 2052 / 2054 / 2056 / 2058 / 2060 / 2061 / 2062 / 2063 / 2064 / 2066 / 5006

Slot: C1+TC1+TCC1

Time: Three Hours

Max. Marks: 100

...NG MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

a) Find the value of k such that the function $f(z) = e^{x}(\cos ky + i \sin ky)$ is analytic. [5]

b) Show that the function $f(x) = \begin{cases} \dfrac{xy^2(x+iz)}{x^2+y^4} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ is not analytic at origin although $C-R$ equations are satisfied at the origin. [5]

In a two dimensional flow of a fluid, the stream function is $\psi(x,\ y) = x^2 - y^2 + \dfrac{x}{x^2+y^2}$, find the complex potential function $f(z) = \phi(x,\ y) + i\psi(x,\ y)$ in terms of z and also find the velocity function $\phi(x,\ y)$.

Obtain the image of the circle $|z-1| = 1$ in the z-plane under the transformation $w = z^2$.

Obtain the bilinear transformation which maps the points $-i,\ 0,\ i$ in the z-plane into the points $-1,\ i,\ 1$ into the w-plane and hence find the invariant points this bilinear transformation.

Obtain the Taylor's series expansion of the function $f(z) = \dfrac{z^2-1}{z^2+5z+6}$ about $z = 1$ and $z = -1$. NO?

Evaluate $\displaystyle\int_0^{\pi} \frac{1}{4+\sin^2\theta}\, d\theta$ using residue theorem.

a) Is $W = \{(a,\ b,\ c,\ d) \mid a = 2b,\ b - 2c + d = 0\}$ a subspace of a vector space $R^4(R)$, where R is the set of all real numbers. If it is a subspace of $R^4(R)$, find the basis and dimension of W. [5]

b) Find the basis and dimension of the null space of $A = \begin{pmatrix} 2 & 0 & 4 & -2 \\ 1 & 3 & -1 & 0 \\ 0 & -2 & 1 & 1 \end{pmatrix}$. [5]