VIT
Vellore Institute of Technology

Final Assessment Test – November/December 2023

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2007 / 2008 / 2009 / 2010 / 8706 Slot: D2+TD2+TD

Time: Three Hours Max. Marks: 1

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION, IS TREATED AS EXAM MALPRACTI

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. If $\psi = (xy)(x^2 - y^2)$, represent the stream function in two dimensional fluid flow, find the corresponding velocity function ϕ and also the complex potential for $w = \phi + i\psi$.

2. If $f(z) = u + iv$ is an analytic function such that $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$. Determine $f(z)$.

3. Determine the image of $1 < x < 2$ under the mapping $w = \frac{1}{z}$ and plot the same.

4. Find the bilinear transformation which maps $z = 1,\ i, -1$ respectively onto $w = i,\ 0, -i$. Hence find the fixed points.

5. If $f(z) = f(z) = \frac{1}{(z+1)(z+3)}$ find Laurent's series expansions in
   (i) $|z| < 1$
   (ii) $1 < |z+1| < 2$.

6. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using Contour Integration.

7. Find a basis for the row and column spaces of

$$A = \begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}.$$

8. Let $T: R^4 \to R^3$ be the linear transformation given by the formula
   $T(x_1, x_2, x_3, x_4) = (4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4)$.
   (i) Calculate a basis for $\ker(T)$.
   (ii) Find a basis for $R(T)$.
   (iii) Verify the dimension theorem.

9. Let $T: P1 \to P2$ be a linear transformation. The matrix of T w.r.t. the bases $S_1 = \{v_1, v_2\}$ and $S_2 = \{w_1, w_2, w_3\}$ is

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ -1 & -2 \end{bmatrix}$$

   (i) Find $[T(v_1)]_{S_2}$ and $[T(v_2)]_{S_2}$
   (ii) Find $T(v_1)$ and $T(v_2)$.
   (iii) Find $T(a_0 + a_1 x)$.
   (iv) Calculate $T(2x + 1)$.

10. Use the Gram–Schmidt process to transform the basis $\{1, x, x_2\}$ of P_2 into an orthonormal basis if

(i) $\langle p, q\rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$

(ii) $\langle p, q\rangle = \int_0^2 p(x)q(x)\,dx$

11. Find the Eigenvalues and the corresponding Eigenvectors of the matrix

$$A=\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}.$$

12. Solve the following system by using the Gauss-Jordan elimination method.

$$x + y + 2z = 1$$
$$2x - y + w = -2$$
$$x - y - z - 2w = 4$$
$$2x - y + 2z - w = 0.$$

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