



## SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –I

Programme Name & Branch: B.Tech All Branches

Slot: A2+TA2+TAA2

Course Name & code: Differential Equations and Transforms BMAT102L

Exam Duration: 90 Min.

Maximum Marks: 50

Answer ALL Questions (5X10=50)

1. Solve the following equation using method of variation of parameters **10 marks**  
$$\frac{d^2y}{dx^2} + y = x \sin x$$
2. Solve the equation  $(x^2D^2 - 2xD + 4)y = x^2 + 2 \log x$  **10 marks**
3. A mass of  $m = 10 \text{ kg}$  is attached to a spring having spring constant  $k = 140 \text{ N/m}$ . The mass is started in motion from the equilibrium position with a velocity of  $1 \text{ m/sec}$  in the upward direction and with an applied external force  $F(t) = 5 \sin t$ . Find the subsequent motion of the mass if the damping constant is  $c = 90 \text{ N.sec/m}$ . **10 marks**
4. a) Form the partial differential equation by eliminating the arbitrary function **5 marks**  
from  $z = f\left(\frac{y}{x}\right)$   
b) Find the complete and singular solutions of the partial differential equation **5 marks**  
 $z = px + qy + 2\sqrt{pq}$
5. Find the general solution of the partial differential equation **10 marks**  
 $(y + z) \frac{\partial z}{\partial x} + (z + x) \frac{\partial z}{\partial y} = x + y$

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# BMAT102L - DIFFERENTIAL EQUATIONS & TRANSFORMS

A2 + TA2 + TAA2

CAT-I - ANSWER KEY

1)  $\frac{d^2y}{dx^2} + y = x \sin x$  (Variation)

aux:  $m = \pm i$  C.F =  $C_1 \cos x + C_2 \sin x$

$W = 1$ ;  $y_1 = \cos x$ ;  $y_2 = \sin x$ .

P.I =  $-y_1 \int \frac{y_2 x}{W} dx + y_2 \int \frac{y_1 x}{W} dx$

=  $-\cos x \int x \sin^2 x dx + \sin x \int x \sin x \cos x dx$

=  $-\cos x \int x (1 - \cos 2x) dx + \sin x \int x \frac{\sin 4x}{2} dx$

=  $-\frac{x^2}{2} \frac{\cos x}{2} + \frac{x}{4} \sin x + \frac{1}{8} \cos x$ .

2)  $(x^2 D^2 - 2xD + 4)y = x^2 + 2 \log x$

put  $x = e^t \Rightarrow \log x = t$

$(D^2 - 3D + 4)y = e^{2t} + t$

C.F  $e^{3/2 t} \left[ C_1 \cos \frac{\sqrt{7}}{2} t + C_2 \sin \frac{\sqrt{7}}{2} t \right]$

P.I =  $\frac{e^{2t}}{2} + \frac{1}{2} (t + 3/4)$

$\therefore y = \left[ C_1 \cos \left( \frac{\sqrt{7}}{2} \log x \right) + C_2 \sin \left( \frac{\sqrt{7}}{2} \log x \right) \right] x^{3/2} + \frac{x^2}{2} + \frac{1}{2} (\log x + 3/4)$

3)  $\frac{d^2x}{dt^2} + 9 \frac{dx}{dt} + 14x = \frac{1}{2} \sin t$

aux:  $m = -2, -7$  C.F =  $C_1 e^{-2t} + C_2 e^{-7t}$

P.I =  $\frac{13}{500} \sin t + \frac{9}{500} \cos t$

Solu is:

$$x(t) = c_1 e^{2t} + c_2 e^{-7t} + \frac{13}{500} \sin t - \frac{9}{500} \cos t$$

Ques:  $x(0) = 0 \Rightarrow c_1 + c_2 - \frac{9}{500} = 0$

$$x'(0) = -1 \text{ (initial vel. in upper direction)}$$

$$2c_1 + 7c_2 = \frac{513}{500}$$

Solve:  $c_1 = -\frac{90}{500}$  ;  $c_2 = \frac{99}{500}$

$$x = \frac{1}{500} [-90 e^{-2t} + 99 e^{-7t} + 13 \sin t - 9 \cos t]$$

④ a)  $px + qy = 0$

b)  $z = px + qy + 2\sqrt{pq}$

Complete soln:  $z = ax + by + 2\sqrt{ab}$

Singular soln:  $z + xy = 0$

⑤  $(y+z)p + (z+x)q = x+y$

$$\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$$

Consider:  $\frac{dx-dy}{y-x} = \frac{dy-dz}{z-y} \Rightarrow \boxed{\frac{x-y}{y-z} = C_1}$

Carry  $\frac{dx-dy}{y-x} = \frac{dx+dy+dz}{x+y+z}$

$$\Rightarrow x-y \sqrt{x+y+z} = C_2$$

$$\therefore y = \varphi(C_1, C_2) = 0$$