SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –I

Programme Name & Branch : **B Tech**

Slot: **B2+TB2+TBB2**

Course Name & code: **Differential Equations and Transforms & BMAT102L**

Class Number (s): VL2023240501562/1540/1550/1551/1536

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Answer ALL Questions

(Only a calculator is to be permitted)

Q.No.	Questions	Max Marks	Course Outcome	Bloom's Taxonomy
1.	Solve the differential equation by the method of undetermined coefficients $$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x+\frac{1}{x}$$	10	CO1	BL2
2.	Solve by the method of variation of parameters $$\frac{d^2y}{dx^2}+y=sec^2x$$	10	CO1	BL2
3.	A series circuit contains a resistor R=24 ohms, an inductor with L= 2 H, a capacitor with C=0.005 F, and a generator producing a voltage of E(t) = 12 sin 10t . Find the charge at time t.	10	CO1	BL3
4.	(i) Form the partial differential equation by eliminating arbitrary function 'f' from $f(z-xy, x^2+y^2)=0$. (ii) Solve $z=px+qy+\left(\frac{q}{p}-p\right)$	5+5	CO1	BL2
5.	Solve $(z^2-2yz-y^2)\,p+(xy+zx)q= xy-zx$	10	CO1	BL2

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KEY

1.

The given equation is

$$(x^2D^2 + 4x\, D + 2)\, y = x + \frac{1}{x}$$

Put $x = e^z$ or $z = \log x$ then (1) becomes

$$(\theta(\theta - 1) + 4\theta + 2)\, y = e^z + e^{-z}$$

The auxiliary equation is $m^2 + 3m + 2 = 0 \Rightarrow m = -1, -2$

So the complimentary function is $\text{C.F.} = C_1 e^{-z} + C_2 e^{-2z}$

Let the trail solution be $y = Ae^z + Bze^{-z}$.

Now,

$$Ae^z - 2Be^{-z} + Bze^{-z} + 3Ae^z + 3Be^{-z} - 3Bze^{-z} + 2Ae^z + 2Bze^{-z} = e^z + e^{-z}$$

Comparing we have, A=1/6 ; B=1.

Hence the solution is

$$y = C_1 e^{-z} + C_2 e^{-2z} + \frac{e^z}{6} + ze^{-z}, \text{ where } z = \log x$$

$$= \frac{C_1}{x} + \frac{C_2}{x^2} + \frac{x}{6} + \frac{\log x}{x}$$

2.

The auxiliary equation is $m^2 + 1 = 0 \Rightarrow m = \pm i$

So the complimentary function is

$$\text{C.F.} = C_1 \cos x + C_2 \sin x$$

Let $f(x) = \cos x$ and $g(x) = \sin x$. The Wronskian of f and g is

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

Let the solution be $y = C_1 \cos x + C_2 \sin x$

where

$$C_1 = -\int \sec^2 x \cdot \sin x \, dx = -\sec x + A$$

and $$C_2 = \int \sec^2 x \cdot \cos x \, dx = \log(\sec x + \tan x) + B$$

Hence the solution is

$$y = (-\sec x + A)\cos x + (\log(\sec x + \tan x) + B)\sin x$$

$$= A\cos x + B\sin x - 1 + \sin x \log(\sec x + \tan x).$$

3. The differential equation for the LCR circuit is governed as

$$L\frac{d^2Q}{dt^2} + R\,\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

$$(i.e)\ (D^2 + 12\,D + 100)\,Q = 6\sin 10t$$

Therefore, the complementary solution is ,

$$Q_h(t) = e^{-6t}\,(A\,\cos 4t + B\,\sin 4t)$$

Let the trail solution be

$$Q_p(t) = C\,\cos 10t + D\,\sin 10t$$

Now,

$$-100\,C\,\cos 10t - 100D\sin 10t - 120C\sin 10t + 120D\cos 10t + 100C\cos 10t + 100D\sin 10t = 6\sin 10t$$

Which implies that $= \frac{-1}{20}$; D=0 .

Therefore , the charge is $Q(t) = e^{-6t}\,(A\,\cos 8t + B\,\sin 8t) - \frac{\cos 10t}{20}$.

4.

(i) $$f(z - xy, x^2 + y^2) = 0 \quad (1)$$

By putting $z - xy = u$ and $x^2 + y^2 = v$, (1) becomes

$$f(u, v) = 0 \quad (2)$$

Differentiating (2) partially with respect to x and then with respect to y, we have

$$\frac{\partial f}{\partial u}.(p - y) + \frac{\partial f}{\partial v}(2x) = 0 \quad (3)$$

and $$\frac{\partial f}{\partial u}(q - x) + \frac{\partial f}{\partial v}(2y) = 0 \quad (4)$$

Eliminating $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ from (3) and (4), we get

$$\begin{vmatrix} p-y & 2x \\ q-x & 2y \end{vmatrix} = 0$$

i.e.
$$2y(p-y) - 2x(q-x) = 0$$

or
$$yp - xq = y^2 - x^2$$

(ii)

The given equation

$$z = px + qy + \left(\frac{q}{p} - p\right) \qquad (1)$$

is a Clairaut's type equation.

∴ The complete solution of (1) is

$$z = ax + by + \frac{b}{a} - a \qquad (2)$$

The general solution of (1) is found out as usual.

To find the singular solution of (1), we differentiate (2) partially with respect to a and then b.

We get

$$0 = x - b/a^2 - 1 \qquad (3)$$

and
$$0 = y + 1/a \qquad (4)$$

Using $a = -\frac{1}{y}$ got from (4) in (3), we get

$$x - by^2 - 1 = 0$$

i.e.
$$b = \frac{x-1}{y^2}$$

Using these values of a and b in (2), we get

$$z = -x/y + \frac{x-1}{y} - \left(\frac{x-1}{y}\right) + \frac{1}{y}$$

i.e. $yz = 1 - x$, which is the singular solution of (1).

5.

This is a Lagrange's linear equation with $P = z^2 - 2yz - y^2$; $Q = xy + zx$; $R = xy - zx$.
The Subsidiary equations are

$$\frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{xy + zx} = \frac{dz}{xy - zx}$$

Comparing the last two ratios, we have

$$y\,dy - d(yz) - z\,dz = 0$$

Integrating we get,

$$y^2 - 2yz - z^2 = a$$

Using the multipliers x,y,z each of the above ratios = $\frac{x\,dx + \;\; dy + z\,dz}{0}$

after integrating, we get, $x^2 + y^2 + z^2 = b$

Therefore the general solution of the given equation is $f(y^2 - 2yz - z^2, x^2 + y^2 + z^2) = 0$.

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