WINTER SEMESTER 2023-2024

SCHOOL OF ADVANCED SCIENCES

DEPARTMENT OF MATHEMATICS

CONTINUOUS ASSESSMENT TEST – I

Course Code: BMAT102L
Course Name: Differential Equations and Transforms
Slot : C2+TC2
Duration : 90 Minutes **Max. Marks: 50**

Answer ALL the following questions.

Q. No.	Question	Marks
1.	Solve the differential equation by method of variation of parameter: $$\frac{d^2y}{dt^2} + 2\,\frac{dy}{dt} + y = e^{-t}\log t$$	10
2.	The charge $q(t)$ on the capacitor at any time t in the LCR circuit, with the resistance $R = 6\Omega$, inductor $L = 0.5$ H, capacitance $C = 0.02\,F$ and e.m.f. $E(t) = 24\sin 10t$. If the initial current and charge on the capacitor are both zero, find charge $q(t)$ and current $i(t)$ at time t.	10
3.	Solve the differential equation : $(x^2D^2 + xD + 1)y = \log x \sin \log x$	10
4.	(i). Form partial differential equation by eliminating arbitrary function: $$xyz = f(x + y + z)$$ (ii). Find the complete integral of partial differential equation: $$9(\mathrm{p}^2\mathrm{z} + \mathrm{q}^2) = 4$$	5+5
5.	Solve the partial differential equation: $$x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$$	10

1)

$$y'' + 2y' + y = e^{-t} \log t$$

$$y_c = c_1 e^{-t} + c_2 t e^{-t}$$

$$W = e^{-2t}$$

$$y_p = -\frac{1}{2} t^2 e^{-t} \log t + \frac{1}{4} t^2 e^{-t} + t^2 e^{-t} \log t - t^2 e^{-t}$$

$$y = y_c + y_p$$

$$= c_1 e^{-t} + c_2 t e^{-t} + \frac{1}{2} t^2 e^{-t} \log t - \frac{3}{4} t^2 e^{-t}$$

2.)

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E(t)$$

$R = 6\Omega$, $L = 0.5H$, $C = 0.02F$

$E(t) = 24 \sin 10t$.

Sol^n - $$0.5 q'' + 6q' + \frac{q}{0.02} = 24 \sin 10t$$

$$(D^2 + 12D + 100) q = 48 \sin 10t$$

$$C.F = y_c = e^{-6t} (c_1 \cos 8t + c_2 \sin 8t)$$

let

$$q_p = A \sin 10t + B \cos 10t$$

By method of undetermined coeff.

$A = 0, \quad B = -2/5$

$$q = e^{-6t}(c_1 \cos 8t + c_2 \sin 8t) - \frac{2}{5}\cos 10t$$

$$i = \frac{dq}{dt} = -6e^{-6t}(c_1 \cos 8t + c_2 \sin 8t) + e^{-6t}(-8c_1 \sin 8t + 8c_2 \cos 8t) + \frac{2}{5} \times 10 \sin 10t$$

$$= e^{-6t}[(-6c_1 + 8c_2)\cos 8t - (6c_2 + 8c_1)\sin 8t] + 4\sin 10t$$

at $t = 0, \; q = 0, \; i = 0$

$c_1 = 2/5, \quad c_2 = 3/10$

$$q(t) = e^{-6t}\left(\frac{2}{5}\cos 8t + \frac{3}{10}\sin 8t\right) - \frac{2}{5}\cos 10t$$

$$i(t) = -5e^{-6t}\sin 8t + 4\sin 10t$$

3.) $(x^2D^2 + xD + 1)y = \log x \sin \log x$

$x = e^z$ s.t. $z = \log x$

$(\theta(\theta-1) + \theta + 1)y = z \log z$

A.E : $\theta^2 + 1 = 0$

$\theta = \pm i$

$y_c = c_1 \cos z + c_2 \sin z$

$$y_p = \frac{1}{4}\left(\frac{-iz^2+z}{\cos z + i\sin z}\right)$$

$$y = c_1 \cos \log x + c_2 \sin \log x + \frac{1}{4} \log x \sin(\log x)$$
$$- \frac{1}{4} (\log x)^2 \cos(\log x)$$

4 I $xyz = f(x+y+z)$

diff w.r.t x , & y

$$yz + xy\left(\frac{\partial z}{\partial x}\right) = f'(x+y+z)(1+p)$$

$$zx + xy(q) = f'(x+y+z)(1+q)$$

$$\frac{yz + xyp}{zx + xyq} = \frac{1+p}{1+q}$$

$$\Rightarrow \boxed{px(y-z) + qy(z-x) = z(x-y)}$$

II $q(p^2z + q^2) = 4$

put $z = f(x+ay)$ soln

& $u = x+ay$ be trim

$f(p, q, z) = 0$

$p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot 1$

$q = \frac{\partial z}{\partial y} = a\frac{\partial z}{\partial u}$

$f\left(\frac{\partial z}{\partial u}, a\frac{\partial z}{\partial u}, z\right) = 0$

put $= 9\left[\left(\frac{\partial z}{\partial u}\right)^2 z + a^2\left(\frac{\partial z}{\partial u}\right)^2\right] = 4$

$= 9\left(\frac{\partial z}{\partial u}\right)^2\left[(z+a^2)\right] = 4$

$= 3\sqrt{z+a^2}\,\frac{\partial z}{\partial u} = 2$

$\int \frac{3}{2}(z+a^2)^{1/2}\,dz = \int du$

$\frac{3}{2}\,\frac{(z+a^2)^{3/2}}{\cancel{3/2}} = u + c$

$(z+a^2)^{3/2} = x+ay+c$

(or) $\boxed{(z+a^2)^3 = (x+ay+c)^2}$

5 (a)

$$x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$$

$$Pp + Qq = R$$

Soln

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{y^2+z - (x^2+z) + (x^2-y^2)} = \frac{dx/x + dy/y + dz/z}{0}$$

$$\log x + \log y + \log z = \log c_1$$

$$\boxed{xyz = c_1}$$

$$= \frac{x\,dx + y\,dy - dz}{x^2(y^2+z) - y^2(x^2+z) - z(x^2-y^2)}$$

$$= \frac{x\,dx + y\,dy - dz}{0}$$

$$\Rightarrow \quad x\,dx + y\,dy - dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} - z = C_2$$

$$\Rightarrow \boxed{x^2 + y^2 - 2z = C_2}$$

$\Rightarrow$ General sol^n

$$f\left(xyz,\ x^2+y^2-2z\right) = 0$$