VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –I

Programme Name & Branch : B.Tech

Slot: D2+TD2

Course Name & code: Differential Equations & Transforms

Class Number (s): VL2023240501538, VL2023240501587, VL2023240501586

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Answer ALL Questions

Q.No.	Question	Max Marks	CO	BL
1.	By method of variation of parameter solve $\frac{d^2y}{dx^2}-12\frac{dy}{dx}+32y=x+e^x$	10	CO1	BL2
2.	Solve $\left(x^2D^2-8xD+20\right)y=\sin(\log x)+x$	10	CO1	BL2
3.	A 2-kg mass is attached to a spring with spring constant 24 N/m. The system is then immersed in a medium imparting a damping force equal to 16 times the instantaneous velocity of the mass. If it is released from rest with downward velocity 1m/sec & if the mass is set in motion with the imposed external force $F(t) = 12\sin 2t$, find the motion of the mass. Finally interpret the solution physically.	10	CO1	BL3
4.	i) Derive partial differential equation from the following function by eliminating the function φ $\varphi\left(x+y+z,x^2+z^2+y^2\right)=0$ ii) Solve $p(1-q^2)=q(1-z)$.	5+5	CO1	BL2
5.	Find the general solution of $p+3q=5z+\tan(y-3x)$	10	CO1	BL2

BMAT102L DE4T CAT-I

Ans ① Method of Variation of parameter

$$\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 32y = x + e^x$$

roots $m = 8, 4$, $CF = c_1 e^{8x} + c_2 e^{4x}$

$$W = \begin{vmatrix} e^{8x} & e^{4x} \\ 8e^{8x} & 4e^{4x} \end{vmatrix} = -4e^{12x} \neq 0$$

$$P.I. = \frac{x}{32} + \frac{3}{256} + \frac{e^x}{21}$$

$$y = c_1 e^{8x} + c_2 e^{4x} + \frac{x}{32} + \frac{3}{256} + \frac{e^x}{21}$$

Ans ② $(x^2 D^2 - 8xD + 20)y = \sin(\log x) + x$

Let $x = e^z$, $\log x = z$

$$(D_1^2 - 9D_1 + 20)y = \sin z + e^z$$

$m = 5, 4$ $C.F. = A_1 e^{5z} + B_1 e^{4z}$

$$P.I. = \frac{x}{12} + \frac{1}{442}\left[9\cos(\log x) + 19\sin(\log x)\right]$$

$y = \cancel{A_1 e^{5z} + B_1 e^{4z} +} = A_1 x^5 + B_1 x^4 + \frac{x}{12} + \frac{1}{442}\left[9\cos(\log x) + 19\sin(\log x)\right]$

Ans 3 Given $m = 2$, $k = 24$, $\beta = 16$, $F(t) = 12\sin 2t$

$x'(0) = 1$, $x(0) = 0$

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = 12\sin 2t$$

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 12x = 6\sin 2t$$, roots $= -6, -2$

$$x = A_1 e^{-6t} + A_2 e^{-2t} - \frac{3}{20}\left[2\cos 2t - \sin 2t\right]$$

$x(0) = A_1 + A_2 - \frac{3}{20}(2)$ $\Rightarrow A_1 + A_2 = \frac{3}{10}$

$x'(0) = 1$ $\Rightarrow 3A_1 + A_2 = -\frac{7}{20}$

$A_1 = -\frac{13}{40}$, $A_2 = \frac{25}{40}$

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$$x = -\frac{13}{40} e^{-6t} + \frac{25}{40} e^{-2t} - \frac{3}{20}[2\cos 2t - \sin 2t]$$

Physical Interpretation

$$\text{root} = \frac{-8 \pm \sqrt{64 - 4\times 12}}{2} = \frac{-8 \pm \sqrt{16}}{2} \to \text{over damped}$$

Q(4) (i) $\phi(x+y+z,\ x^2+z^2+y^2) = 0$

Diffn. w.r.t. x & y

$$(1+p)(y + zq) = (1+q)(x + zp)$$

$$p(y-z) + q(z-x) = x-y$$

(ii) Solve $p(1-q^2) = q(1-z)$

Let $q = ap$

$$\cancel{p}(1-a^2p^2) = a\cancel{p}(1-z)$$

$$p^2 = \frac{1-a+az}{a^2} \Rightarrow p = \frac{1}{a}\sqrt{1-a+az}$$

$$dz = p\,dx + q\,dy$$

$$\frac{a\,dz}{\sqrt{1-a+az}} = dx + a\,dy$$

$$\boxed{2\sqrt{1-a+az} = x + ay + c_1}$$

Q(5) $p + 3q = 5z + \tan(y-3x)$

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y-3x)}$$

$$\frac{dx}{1} = \frac{dy}{3} \Rightarrow \boxed{y - 3x = c_1}$$

$$\frac{dy}{3} = \frac{dz}{5z + \tan c_1} \Rightarrow \frac{y}{3} - \frac{1}{5}\log\{5z + \tan(y-3x)\} = c_2$$

$$\therefore F(c_1, c_2) = 0$$