VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act 1956)

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –II

Programme Name & Branch : B.Tech

Slot: C2+TC2+TCC2

Course Name & code: Differential Equations and Transforms & BMAT102L

Class Number (s): VL202324501532, 1531, 1528, 4954, 1582

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Students are permitted to bring one text book/ hand written note book only.

ANSWER ALL THE QUESTIONS

Q.No.	Question	Max Marks	CO	BL
1.	Express the function $f(t)=\begin{cases} \frac{3t}{2}, & 0\le t<2 \\ 6-\frac{3t}{2}, & 2\le t<4 \\ 0, & t\ge 4 \end{cases}$ in terms of unit step function and hence find the Laplace transform of *f(t)*.	10	CO2	BL2
2.	Find the inverse Laplace transform of $\left(\frac{10}{(s+1)(s^2+4)}\right)$ using convolution theorem.	10	CO2	BL2
3.	Solve $\frac{d^2y}{dt^2}+y=3\delta(t-1)+2\delta(t)$ with $y(0)=0, \frac{dy}{dt}(0)=0$, using Laplace transform.	10	CO4	BL3
4.	Obtain the Fourier series for $f(t)=1+t+t^2$ in the interval $(-\pi,\pi)$. Hence find the sum of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots\ldots\ldots$	10	CO3	BL3
5.	Find the half-range cosine series for the function $f(t)=\begin{cases} \lambda t, & for\ \ 0<t<\frac{1}{2} \\ \lambda(1-t) & for\ \ \frac{1}{2}<t<1 \end{cases}$.	10	CO3	BL2

C_2 slot

1. $$f(t) = \begin{cases} \frac{3t}{2} & ; 0 \le t < 2 \\ 6 - \frac{3t}{2} & , 2 \le t < 4 \\ 0 & , t \ge 4 \end{cases}$$

$$f(t) = [u(t-0) - u(t-2)]\{3t/2\}$$
$$+ [u(t-2) - u(t-4)]\{6 - 3t/2\}$$
$$= u(t-0)\left(\frac{3t}{2}\right) + u(t-2)(6-3t)$$
$$+ u(t-4)(-6 + 3t/2)$$
$$\Rightarrow u(t-0)\left(\frac{3t}{2}\right) - 3u(t-2)(t-2)$$
$$+ 3u(t-4)(t-4)/2$$

$$L\{f(t)\} = L\left\{u(t-0)\frac{3t}{2}\right\} - 3L\{u(t-2)(t-2)\}$$
$$+ 3L\left\{u(t-4)\frac{(t-4)}{2}\right\}$$
$$= \frac{3}{2s^2} - 3\left(\frac{e^{-2s}}{s^2}\right) + \frac{3}{2}\left(\frac{e^{-4s}}{s^2}\right)$$
$$= \frac{3}{2}\frac{(1 - 2e^{-2s} + e^{-4s})}{s^2} = \frac{3}{2}(1 - e^{-2s})^2$$

(2) $L^{-1}\left[\dfrac{10}{(s+1)(s^2+4)}\right]$

$\Rightarrow 10\, L^{-1}\left[\dfrac{1}{s+1}\right] \cdot L^{-1}\left[\dfrac{1}{s^2+4}\right] = 10 \cdot e^{-t} * \dfrac{1}{2}\sin 2t$

$\Rightarrow 5\int_0^t e^{-u} \cdot \cancel{t}\, \sin 2(t-du) \,.\, du$

$\Rightarrow$ (or) $5\int_0^t \sin 2u \,.\, e^{-(t-u)}\, du$

$\Rightarrow 5e^{-t}\int_0^t e^{u} \sin 2u\, du$

$\Rightarrow 5e^{-t}\left[\dfrac{e^{t}}{1+4}\left(\sin 2t - 2\cos 2t + 2\right)\right]$

$\Rightarrow \sin 2t - 2\cos 2t + 2e^{-t}$

(3) $y'' + y = 2\delta(t) + 3\delta(t-1) \;;\; y(0) = 0 = y'(0)$

$L\{y''\} + L\{y\} = 2L\{\delta(t)\} + 3L[\delta(t-1)]$

$s^2\bar{y}(s) - y(0) + \bar{y}(s) = 2 + 3e^{-s}$

$\bar{y}(s)(s^2+1) = 2 + 3e^{-s}$

$\bar{y}(s) = \dfrac{2}{s^2+1} + 3\dfrac{e^{-s}}{s^2+1}$.

(4)

ILT

$$L\{\bar{y}(s)\} = 2L^{-1}\left\{\frac{1}{s^2+1}\right\} + 3L^{-1}\left\{\frac{e^{-s}}{s^2+1}\right\}$$

$$y(t) = 2.\sin t + 3\sin(t-1)u(t-1)$$

$$\therefore y(t) = \begin{cases} 2\sin t & \text{if } 0 \le t < 1 \\ 2\sin t + 3\sin(t-1) & \text{if } t \ge 1 \end{cases}$$

(4) $l = \pi$

$$a_0 = \frac{1}{\pi}\left[\frac{2\pi^3}{3} + 2\pi\right]$$

$$a_n = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{2(-1)^{n+1}}{n}$$

Put $x = \pi$

$$\frac{2\pi^2}{3} + \pi = 4\sum_1^\infty \frac{1}{n^2}$$

$$x = -\pi$$

$$\frac{2\pi^2}{3} - \pi = 4\sum_1^\infty \frac{1}{n^2}$$

$$\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$$

(5) $a_0 = \frac{\lambda}{2}$

$$a_n = \frac{2\lambda}{n^2\pi^2}\left[2\cos\frac{n\pi}{2} - 1 - (-1)^n\right]$$

when n is odd, $\cos\frac{n\pi}{2} = 0$

$a_n = 0$, n is odd.

$$f(t) = \frac{\lambda}{4} - \frac{8\lambda}{\pi^2}\left[\frac{1}{2^2}\cos 2\pi t + \frac{1}{6^2}\cos 6\pi t + \dots\right]$$