VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –II

Programme Name & Branch : B.Tech

Slot: D2+TD2+TDD2

Course Name & code: Differential Equations & Transforms, BMAT102L

Class Number (s): VL2023240501538, VL2023240501587, VL2023240501586

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): **Only hand written notes and text books permitted**
Answer all the questions ($5 \times 10 = 50$ marks)

Q. No.	Question	Max Marks
1.	Find the Laplace transform of (a) $f(t) = t(3\sin 2t - 2\cos 2t)$ (b) $f(t) = \begin{cases} -0.5t + 1, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$	10
2.	Find the inverse Laplace transform of $\frac{S^2}{(S^2+9)(S^2+25)}$ using Convolution theorem.	10
3.	Solve the differential equation $y'' + 5y' + 6y = u(t-1) + \delta(t-2),\ y(0) = 0,\ y'(0) = 1,$ using method of Laplace Transform.	10
4.	Expand $f(x) = x + \|x\|, (-\pi < x < \pi)$ in a Fourier series.	10
5.	Find the half range cosine series of the function $f(x) = \begin{cases} 0, & 0 < x < \pi/2 \\ x, & \frac{\pi}{2} < x < \pi \end{cases}$	10

1 (a) Find Laplace transform of $f(t) = t(3\sin 2t - 2\cos 2t)$

$L\{3\sin 2t - 2\cos 2t) = 3\cdot\frac{2}{s^2+2^2} - 2\cdot\frac{s}{s^2+2^2} = \frac{6-2s}{s^2+4}$

$$L\{t(3\sin 2t - 2\cos 2t)\} = (-1)\frac{d}{ds}\left(\frac{6-2s}{s^2+4}\right) = \frac{8+12s-2s^2}{(s^2+4)^2}$$

(b) $f(t) = \begin{cases} -0.5t+1, & 0 \le t < 1 \\ 0 & t > 1 \end{cases}$

$$L\{f(t)\} = \frac{1}{s} - \frac{0.5e^{-s}}{s} - \frac{0.5}{s^2} + \frac{0.5e^{-s}}{s^2}$$

(Q(2)) $L^{-1}\left\{\frac{s^2}{(s^2+9)(s^2+25)}\right\} = L^{-1}\left(\frac{s}{s^2+5^2}\right) * L^{-1}\left(\frac{s}{s^2+3^2}\right)$

$= \cos 5t * \cos 3t$

$= \frac{1}{16}(5\sin 5t - 3\sin 3t)$

QN(3) $y'' + 5y' + 6y = u(t-1) + \delta(t-2)$, $y(0)=0$, $y'(0)=1$

$L[y''] + 5L[y'] + 6\cdot L[y] = L\{u(t-1)\} + L\{\delta(t-2)\}$

$s^2L[y] - s\,y(0) - y'(0) + 5sL[y] - 5y(0) + 6L[y] = \frac{e^{-s}}{s} + e^{-2s}$

$(s^2+5s+6)L[y] - 1 = \frac{e^{-s}}{s} + e^{-2s}$

$L[y] = \frac{1}{(s+2)(s+3)} + \frac{e^{-s}}{s(s+2)(s+3)} + \frac{e^{-2s}}{(s+2)(s+3)}$

$= \left(\frac{1}{s+2} - \frac{1}{s+3}\right) + e^{-s}\left(\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}\right) + e^{-2s}\left(\frac{1}{s+2} - \frac{1}{s+3}\right)$

$L^{-1}(y) = e^{-2t} - e^{-3t} + \frac{1}{6}u(t-1) + \frac{1}{3}e^{-(t-2)}u(t-2) - \frac{1}{2}e^{-2(t-1)}u(t-1)$

$+ e^{-2(t-2)}u(t-2) - e^{-3(t-2)}u(t-2)$

$\frac{1}{3}e^{-3(t-1)}u(t-1)$

Q(4) Expand $f(x) = x + |x|, \quad -\pi < x < \pi$

$$f(x) = \begin{cases} x + (-x) & \text{if } -\pi < x < 0 \\ x + x & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0\cdot dx + \int_0^{\pi} 2x\,dx\right] = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi} 2x\cos nx\,dx$$

$$= \frac{2}{n\pi}\left[x\sin nx\right]_0^{\pi} + \frac{2}{n^2\pi}\left[\cos \pi x\right]_0^{\pi} = \frac{2\left[(-1)^n - 1\right]}{n^2\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{-2}{n\pi}\left[x\cos nx\right]_0^{\pi} + \frac{2}{n^2\pi}\left[\sin nx\right]_0^{\pi} = \frac{2(-1)^{n+1}}{n}$$

$$f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\left[(-1)^n - 1\right]}{n^2}\cos nx + 2\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin nx$$

Q(5) Half range cosine $f(x) = \begin{cases} 0, & 0 < x < \pi/2 \\ x & \pi/2 < x < \pi \end{cases}$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_0^{\pi/2} 0\cdot dx + \int_{\pi/2}^{\pi} x\cdot dx\right] = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{\pi/2}^{\pi} = \frac{3\pi}{8}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot\cos\left(\frac{n\pi x}{\pi}\right)dx = \frac{2}{\pi}\left[\int_0^{\pi/2} 0\cdot\cos nx\,dx + \int_{\pi/2}^{\pi} x\cdot\cos nx\,dx\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - \cos\frac{n\pi}{2}\right] - \frac{1}{n}\sin\frac{n\pi}{2}$$

$$f(x) = \frac{3\pi}{8} + \sum_{n=1}^{\infty}\left[\frac{2}{n^2\pi}\left\{(-1)^n - \cos\frac{n\pi}{2}\right\} - \frac{1}{n}\sin\frac{n\pi}{2}\right]\cos nx$$

$$= \frac{3\pi}{8} - \left(\frac{2}{\pi} + 1\right)\cos x + \frac{1}{\pi}\cos 2x + \left(\frac{1}{3} - \frac{2}{9\pi}\right)\cos 3x + \cdots$$