VIT
Vellore Institute of Technology

Final Assessment Test – June 2023

Course: BMAT102L - Differential Equations and Transforms

Class NBR(s): 0377 / 0398 / 4426 / 4459 / 4500 / 4502 / 4587 / 4589 / 4591 / 4660 / 4662 / 4664 / 4868 / 4870 / 4872 / 4874 / 4876 / 4878 / 4898 / 4900 / 4902 / 4904 / 4906 / 04908

Slot: A1+TA1+TAA1

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions
(10 X 10 = 100 Marks)

1. The radial displacement u in a rotating disc at a distance r from the axis is given by the linear differential equation with variable coefficients
$u = r\frac{d}{dr}\left(r\left(\frac{du}{dr}\right)\right) + ar^3$, where 'a' is a constant . Solve the equation.

2. A condenser of capacity C discharged through an inductance and resistance R in series and the charge q at time t satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E$. Given that $L = 0.25$ Henries, $R = 12.5$ Ohms, $C = 10^{-2}$ Farads, $E = 3\sin 2t$, and that when $t = 0$, charge in the capacitor and current in the circuit are zero. Find the charge q in terms of t by using method of variation of parameters.

3. (i) Form the partial differential equation by eliminating the arbitrary functions from $z = \frac{1}{x}[f(x-ay) + g(x+ay)]$.
(ii) Obtain the complete solution of the partial differential equation $p(1+q^2) = q(z-a)$.

4. Solve the partial differential equation $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = (x+y)^2 z$.

5. Find the Laplace transform of $f(t) = \begin{cases} 2t & 0<t<2 \\ 8-2t & 2<t<4 \end{cases}$ where $f(t) = f(t+4)$ and also draw the function $f(t)$.

6. Find the inverse Laplace transform of the function $Y(s) = \frac{a(s^2-a^2)}{s^4+4a^4}$.

7. Solve $\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial t} = 2x$ where $x > 0, t > 0, u(x,0) = 1, u(0,t) = 1$ by using Laplace Transforms.

8. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $(0,3)$ and deduce that the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + - - -$.

9. Obtain the Fourier series for $f(x) = |x|$ in $-\pi < x < \pi$ and also by using Parseval's identity formula, evaluate $1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$.

10. Find the Fourier cosine transform of $f(x) = \frac{1}{x^2+25}$ and hence derive Fourier sine transform of $f(x) = \frac{x}{x^2+25}$.

11. If $U(z) = \frac{2z^2+3z+12}{(z-1)^4}$ then find the value of u_2 and u_3.

12. Solve the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 36$ given that $y_0 = y_1 = 0$ using Z–Transforms.

⇔⇔⇔