

**VIT**

Vellore Institute of Technology

**Final Assessment Test - June 2023**Course: **BMAT102L**

- Differential Equations and Transforms

Class NBR(s): 0399 / 4893 / 4895 / 4897

Slot: B2+TB2+TB

Time: Three Hours

Max. Marks: 100

**KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MALPRACTICE**Answer any **TEN** Questions

(10 X 10 = 100 Marks)

1. Find the general solution of the second order differential equation ✓

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \left(\frac{\log x}{x}\right)^2$$

2. An RCL circuit connected in series has a resistance of 5 ohms, an inductance of 0.05 henry, a capacitor of  $4 \times 10^{-4}$  farad, and an applied alternating emf of  $200 \cos(100t)$  volts. Find an expression for the current flowing through this circuit if the initial current and the initial charge on the capacitor are both zero. ✓

3. Find the complete integral of the equation  $z^2 = p^2 + q^2 + 1$ . ✓

4. Solve the equation  $(3z - 4y)p + (4x - 2z)q = 2y - 3x$ . ✓

5. Find the Laplace transform of the square wave function

$$f(t) = \begin{cases} k, & 0 \leq t \leq a \\ -k, & a \leq t \leq 2a \end{cases} \text{ with } f(t+2a) = f(t) \text{ for all } t. \checkmark$$

6. Use convolution theorem to evaluate the integral

$$\int_0^t u^2 e^{-a(t-u)} du.$$

7. Using Laplace transform, solve the differential equation

$$x''(t) + 3x'(t) + 2x(t) = H(t-2) \text{ with } x(0) = x'(0) = 0. \checkmark$$

8. Find the Fourier series expansion of  $f(x)$  given by  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$  and also find the value of  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ .

9. Find the half range sine series for  $f(x) = \begin{cases} x & \text{in } (0, \frac{\pi}{2}) \\ \pi - x & \text{in } (\frac{\pi}{2}, \pi) \end{cases}$  in  $(0, \pi)$  and deduce that the sum of the series  $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ . ✓

10. Find the Fourier transform of  $f(x) = \begin{cases} 1-x^2, & \text{in } |x| \leq 1 \\ 0 & \text{in } |x| > 1 \end{cases}$ . Also prove that

$$\int_0^{\infty} \frac{\sin s - s \cos s}{s^3} \cos \frac{s}{2} ds = \frac{3\pi}{16}.$$

11. Using convolution theorem find  $Z^{-1} \left\{ \frac{z^4}{(z-2)^2(z-3)} \right\}$ .

12. Using Z-transform solve  $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$  with  $y_0 = y_1 = 0$ . ✓

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