VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF MECHANICAL ENGINEERING
CONTINUOUS ASSESSMENT TEST – I
WINTER SEMESTER 2023-2024

Reg. No.
(To be filled by the student)

Programme Name & Branch : **B.Tech, Mechanical Engineering**
Course Code : **BMEE215L**
Course Name : **Engineering Optimization**
Faculty Name(s) : **Dr. Rajyalakshmi G, Dr. Dega Nagaraju, Dr. Raviteja Buddala, Dr. Sudhakara Pandian R, Dr. Sivaprasad Darla**
Class Number(s) : **VL2023240504643, 649, 651, 645, 652**
Slot : **C1+TC1+TCC1**
Date of the Exam : **13-02-2023 (9.30 AM-11.00 AM)**
Exam Duration : **90 minutes** **Maximum Marks: 50**

General instruction(s): Answer ALL Questions

Q. No	Question	Marks	Course Outcome (CO)	Bloom's Taxonomy (BL)
1.	Professor Hamer is on a low cholesterol diet. During lunch at the college cafeteria, he always chooses between two meals, Pasta or Tofu. The table below lists the amount of protein, carbohydrates, and vitamins each meal provides along with the amount of cholesterol he is trying to minimize. Mr. Hamer needs at least 200 grams of protein, 960 grams of carbohydrates, and 40 grams of vitamins for lunch each month. Over this time period, how many days should he have the Pasta meal, and how many days the Tofu meal so that he gets the adequate amount of protein, carbohydrates, and vitamins and at the same time minimizes his cholesterol intake? Formulate the problem in LPP form and find the optimal solution using Graphical Approach. **PASTA** / **TOFU** PROTEIN: 8g / 16g CARBOHYDRATES: 60g / 40g VITAMIN C: 2g / 2g CHOLESTEROL: 60mg / 50mg	10	1	BL2
2.	Solve the following LPP using simplex method and find the optimal solution MAX $Z=3X_1+4X_2$ Subject to $X_1- X_2\leq 1$ $-X_1+X_2\leq 2$ $X_1, X_2 \geq 0$	10	1	BL2
3.	Solve the following LPP using Big M method Min $Z=600X_1+500X_2$ Subject to $2X_1+X_2\leq 80$ $X_1+2X_2\geq 60$ $X_1, X_2 \geq 0$	10	1	BL3

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF MECHANICAL ENGINEERING
CONTINUOUS ASSESSMENT TEST – I
WINTER SEMESTER 2023-2024

Reg. No.
(To be filled by the student)

--

<table>
<tr><td>4.</td><td>Find the initial basic feasible solution of the given transportation problem using Vogel's Approximation Method and determine an Optimal solution using MODI method.

<table>
<tr><th>Destination / Source</th><th>D1</th><th>D2</th><th>D3</th><th>D4</th><th>D5</th><th>Capacity</th></tr>
<tr><td>S1</td><td>2</td><td>11</td><td>10</td><td>3</td><td>7</td><td>4</td></tr>
<tr><td>S2</td><td>1</td><td>4</td><td>7</td><td>2</td><td>1</td><td>8</td></tr>
<tr><td>S3</td><td>3</td><td>9</td><td>4</td><td>8</td><td>12</td><td>9</td></tr>
<tr><td>Requirement</td><td>3</td><td>3</td><td>4</td><td>5</td><td>6</td><td>21</td></tr>
</table></td><td>10</td><td>2</td><td>BL3</td></tr>
<tr><td>5.</td><td>Find Solution of Assignment problem using Hungarian method to minimize the cost
<table>
<tr><th>Job\ ⇨
Machine
⇩</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><td>A</td><td>10</td><td>5</td><td>13</td><td>15</td><td>16</td></tr>
<tr><td>B</td><td>3</td><td>9</td><td>18</td><td>13</td><td>6</td></tr>
<tr><td>C</td><td>10</td><td>7</td><td>2</td><td>2</td><td>2</td></tr>
<tr><td>D</td><td>7</td><td>11</td><td>9</td><td>7</td><td>12</td></tr>
<tr><td>E</td><td>7</td><td>9</td><td>10</td><td>4</td><td>12</td></tr>
</table></td><td>10</td><td>2</td><td>BL2</td></tr>
</table>

NOTE*: Please refer below to the BL – Bloom's Taxonomy Levels and report the respective level in the questions for moderation. After the moderation from the school office, kindly remove the COs and BLs in the final copies, which are supposed to be distributed to the students.

Bloom's Taxonomy Levels	Category
BL1	Remembering
BL2	Understanding
BL3	Applying
BL4	Analysing
BL5	Evaluating
BL6	Creating

****

CAT-I -KEY

1. Sol.

Let x = The number of days Mr. Hamer eats Pasta.

and y = The number of days Mr. Hamer eats Tofu.

Since he is trying to minimize his cholesterol intake, our objective function represents the total amount of cholesterol C provided by both meals.

$$C=60x+50y$$

The constraint associated with the total amount of protein provided by both meals is

$$8x+16y\geq 200$$

Similarly, the two constraints associated with the total amount of carbohydrates and vitamins are obtained, and they are

$$60x+40y\geq 960$$
$$2x+2y\geq 40$$

The constraints that state that x and y are non-negative are

$$x\geq 0, \text{ and } y\geq 0.$$

Summary of the given information as follows:

$$\begin{aligned} \textbf{Minimize} \quad & C=60x+50y \\ \textbf{Subject to:} \quad & 8x+16y \geq 200 \\ & 60x+40y \geq 960 \\ & 2x+2y \geq 40 \\ & x\geq 0;\ y\geq 0 \end{aligned}$$

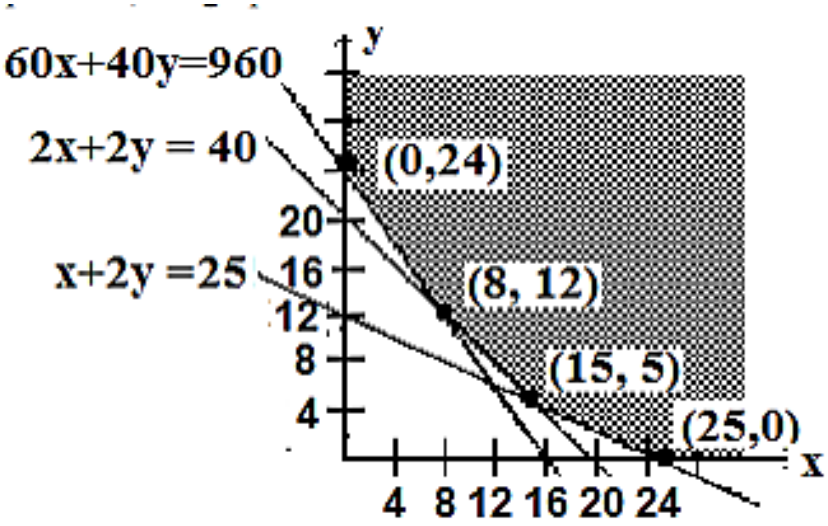
Graphical Approach

Shaded the unbounded feasibility region, where all constraints are satisfied.

To minimize the objective function, we find the vertices of the feasibility region. These vertices are (0, 24), (8, 12), (15, 5) and (25, 0). To minimize cholesterol, we will substitute these points in the objective function to see which point gives us the smallest value. The results are listed below.

Critical points	Income
(0, 24)	60(0) + 50(24) = 1200
(8, 12)	60(8) + 50(12) = 1080
(15, 5)	60(15) + 50(5) = 1150
(25, 0)	60(25) + 50(0) = 1500

The point (8, 12) gives the least cholesterol, which is 1080 mg. This states that for every 20 meals, Professor Hamer should eat Pasta 8 days, and Tofu 12 days.



2. Solve the following LPP

MAX $\mathbf{Z=3X_1+4X_2}$

Subject to $\mathbf{X_1- X_2\leq 1}$ ----- $\mathbf{X_1- X_2+S_1= 1}$

$\mathbf{-X_1+X_2\leq 2}$------$\mathbf{X_1+X_2 +S_2=2}$

$\mathbf{X_1, X_2 \geq 0}$

Standard form

MAX $\mathbf{Z=3X_1+4X_2+0.S_1+0.S_2}$

Subject to $\mathbf{X_1- X_2\leq 1}$ ----- $\mathbf{X_1- X_2+S_1= 1}$ ------ **(1)**

$\mathbf{-X_1+X_2\leq 2}$------$\mathbf{X_1+X_2 +S_2=2}$----- **(2)**

$\mathbf{X_1, X_2 , S_1, S_2 \geq 0}$

Finding Initial Basic feasible solution

Assume $X_1=0$ $X_2=0$; From (1), $S_1=1$

From (2), $S_2=2$

Z=0

	C_j	3	4	0	0		
C_b	Basic	X_1	X_2	S_1	S_2	b	θ
0	S_1	1	-1	1	0	1	-1
0	S_2	-1	1	0	1	2	2
	Zj	0	0	0	0	0	
	C_j-Z_j	3	4	0	0		

Key Row

Key Column

	C_j	3	4	0	0		
C_b	Basic	X_1	X_2	S_1	S_2	b	θ
0	S_1	0	0	1	1	3	3/0=α
4	X_2	-1	1	0	1	2	2/-1=-2
	Zj	-4	4	0	4	8	
	C_j-Z_j	7	0	0	-4		
			Solution is unbounded				

Key Column

3. Solve the following LPP using Big M method

Min $\mathbf{Z=600X_1+500X_2}$

Subject to $\mathbf{2X_1+X_2 \leq 80}$

$\mathbf{X_1+2X_2 \geq 60}$

$\mathbf{X_1, X_2 \geq 0}$

Sol. Standard form

Min $Z = 600X_1 + 500X_2 + 0S_1 + 0S_2 + M A_1$

Subject to $2 X_1 + X_2 + S_1 = 80$

$X_1 + 2 X_2 - S_2 + A_1 = 60$

and $X_1, X_2, S_1, S_2, A_1 \geq 0$

The initial basic feasible solution is obtained by setting $X_1 = X_2 = S_2 = 0$ and S1=80, A1=60 and Z=60M

	C_j	600	500	0	0	M		
C_b	Basic	X_1	X_2	S_1	S_2	A_1	b	θ
0	S_1	2	1	1	0	0	80	80
M	A1	1	2	0	-1	1	60	30
	Zj	M	2M	0	-M	M	60M	
	C_j-Z_j	600-M	500-2M	0	M	0		

	C_j	600	500	0	0		
C_b	Basic	X_1	X_2	S_1	S_2	b	
0	S_1	3/2	0	1	1/2	**50**	
500	X_2	1/2	1	0	-1/2	**30**	
	Zj	250	500	0	-250	**15000**	
	C_j-Z_j	350	0	0	250		

X1=-, X2=30, Z=15000

All the numbers in the Cj – Zj row are either zero or positive and also artificial variables have been reduced to zero, an optimum solution has been arrived at with x1 = 0, x2 = 30 and total minimum cost, Z = 15000.

4. Find the Optimal solution for the given initial feasible allocation of the transportation problem using MODI method.

Destination / Source	D1	D2	D3	D4	D5	Capacity
S1	**2**	**11**	**10**	**3**	**7**	4
S2	**1**	**4**	**7**	**2**	**1**	8
S3	**3**	**9**	**4**	**8**	**12**	9
Requirement	3	3	4	5	6	21

Vogel's Approximation Method

Destination / Source	D1	D2	D3	D4	D5	Penalty	Capacity	
S1	**2**	**11**	**10**	**3 (4)**	**7**	1/1/1/1	4/0	
S2	**1**	**4(2)**	**7**	**2**	**1 (6)**	0/1	8/2/0	
S3	**3 (3)**	**9 (1)**	**4(4)**	**8 (1)**	**12**	1/1/1/5	9/5/2/1	
Penalty	1/1/1/1	5/5/2/2	3/3	1/1/5/5	6			
Requirement	3/0	3/1/0	4/0	5/1	6/0		21	

Initial Solution is

Source\ Destination	D1	D2	D3	D4	D5	Capacity
S1	**2**	**11**	**10**	**3 (4)**	**7**	4
S2	**1**	**4(2)**	**7**	**2**	**1 (6)**	8
S3	**3 (3)**	**9 (1)**	**4 (4)**	**8(1)**	**12**	9
Requirement	3	3	4	5	6	21

Total cost= 3x3 +4x2 +9x1 +4x4 +3x4 +8x1 +1x6 = 68

	V1=-2	V2=4	V3=-1	V4=3		Capacity
U1=0	**2**	**11**	**10**	**3 (4)**	**7**	4
U2=0	**1**	**4(2)**	**7**	**2**	**1 (6)**	8
U3=5	**3 (3)**	**9 (1)**	**4 (4)**	**8(1)**	**12**	9
Requirement	3	3	4	5	6	21

	V1=-2	V2=4	V3=-1	V4=3	V5=1	
U1=0	**2-(-2)=4**	**11-(0+4)=7**	**10-(0-1)=11**	**3 (4)**	**7-(1+0)=6**	
U2=0	**1-(0-2)=3**	**4(2)**	**7-(0-1)=8**	**2-(0+3)=-1**	**1 (6)**	
U3=5	**3 (3)**	**9 (1)**	**4 (4)**	**8(1)**	**12-(5+1)=6**	

	V1=-2	V2=4	V3=-1	V4=3	V5=1	
U1=0				**3 (4)**		
U2=0		**4(2)-**		**2-(0+3)=-1**	**1 (6)**	
U3=5	**3 (3)**	**9 (1)+**	**4 (4)**	**8(1) -**		

	V1=-1	V2=5	V3=0	V4=3	V5=1	
U1=0	**2-(0-1)=3**	**11-5=6**	**10**	**3 (4)**	**7-1=6**	
U2=-1	**1-(-1-1)=3**	**4(1)**	**7-(-1)=8**	**2(1)**	**1 (6)**	
U3=4	**3 (3)**	**9 (2)**	**4 (4)**	**8-7=1**	**12-5=7**	

Total cost= 3x4 +4x1 +2x1 +3x3 +9x2+4x4 +1x6 = 67

5. **Find Solution of Assignment problem using Hungarian method to minimize the cost**

Job\ ⇨ Machine ⇩	1	2	3	4	5
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

Job\ ⇨ Machine ⇩	1	2	3	4	5
A	5	0	8	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

Job\ ⇨ Machine ⇩	1	2	3	4	5	
A	5	0	8	10	11	
B	0	6	15	10	3	✓
C	8	5	0	0	0	
D	0	4	2	0	5	✓
E	3	5	6	0	8	✓
	✓			✓		

Job\ ⇨ Machine ⇩	1	2	3	4	5	
A	7	0	8	12	11	
B	0	4	13	10	1	
C	10	5	0	2	0	
D	0	2	0	0	3	
E	3	3	4	0	6	
Assignment cost= 5+3+2+9+4=23						