



VIT
Vellore Institute of Technology

Final Assessment Test – November/December 2023

Course: **BMAT201L** - Complex Variables and Linear Algebra
 Class NBR(s): 2024 / 2025 / 2026 / 2029 / 2033 / 2035
 Time: Three Hours
 Slot: B2+TB2+TBB2
 Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MALPRACTICE
 Answer any TEN Questions
 (10 X 10 = 100 Marks)

1. Verify that the function $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic and determine its conjugate. Also find $f(Z)$.
2. If $w = \phi + i\psi$ represents the complex potential for an electric field and $\phi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine ψ .
3. Find the image of $|Z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.
4. Find the bilinear transformation which maps the points $2, i, -2$ in the Z -Plane into the points $1, i, -1$ in the w -Plane.
5. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series if
 - (i) $|Z| > 3$
 - (ii) $2 < |Z| < 3$
6. Evaluate $\int_c \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ using residue theorem. Where " c " is the circle $|Z| = 3$.
7. a) Verify that the vectors $(1, 2, 1)$ $(2, 1, 0)$ $(1, -2, 2)$ form a basis for $R^3(R)$ or not. [5]
 b) Find the rank and nullify for the matrix [5]

$$\begin{bmatrix} 1 & -1 & -2 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 4 & 0 & 0 & 1 & -2 \\ 2 & 3 & 8 & -2 & -1 \end{bmatrix}$$
8. Let $T: R^3 \rightarrow R^3$ be a linear transformation defined by

$$T(x, y, z) = (x + 3y - 2z, 2x + 3y, y - z)$$
 Verify whether T is invertible or not? If so find T^{-1} .
9. Obtain the matrix representation of a linear transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (4z, 3x + 5y - 2z, x + y + 4z)$ relative to the basis $B = \{(1, 1, 1)(1, 1, 0)(1, 0, 0)\}$.
10. Apply Gram-Schmidt orthogonalization process to find an orthogonal basis for the subspace spanned by $\{(1, 2, 0, 3)(4, 0, 5, 8)(8, 1, 5, 6)\}$.
11. Compute the Eigen values and corresponding Eigen vectors of $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$.
12. Solve the following system of equations by Gauss Jordan method

$$\begin{aligned} x + y + z &= 9 \\ 2x - 3y + 4z &= 13 \\ 3x + 4y + 5z &= 40. \end{aligned}$$

