VIT
Vellore Institute of Technology

Final Assessment Test – November/December 2023

Course: BMAT203L - Linear Algebra and Differential Equations

Class NBR(s): 2192 / 2193 Slot: C2+TC2+TCC2

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Consider the system of linear equations

$$x + z + 2w = 2$$
$$y - 2z - w = 1$$
$$x - z + w = 2\,.$$

a) Write the augmented matrix for the above system of linear equations in the form of $[A|B]$.

b) Reduce $[A|B]$ into its reduced row echelon form.

c) Check whether the system is consistent. If it is consistent write the solution set.

2. Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix}$.

3. a) Determine whether the sequence $a_n = \cos\left(\frac{n}{2^n}\right)$ is converges or diverges. If it converges find the limit. [5]

b) Check the convergence/divergence of the sequence $\sum_{n=0}^{\infty} \frac{4}{(2n)\,5^n}$. [5]

4. Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n(3x-2)^n}{n+1}$.

5. Find the Maclaurin's series of the function $f(x) = x\,e^{2x}$.

6. Find the solution of the initial value problem $\frac{dy}{dt} + \frac{2y}{t} = t^3$ with the initial condition $y(1) = 1$.

7. Sketch the phase lines for the given differential equation. Identify the equilibrium points as sinks, sources, or nodes.

(i) $\frac{dy}{dt} = y^3 - 4y$.

(ii) $\frac{dy}{dt} = y\cos y\,, -\pi \le y \le \pi$.

8. Consider the following system of differential equations

$$\frac{dx}{dt} = x^2 - 2xy + 2y$$
$$\frac{dy}{dt} = y^2 - xy + 3x.$$

Let $x(0) = 1,\ y(0) = 1$. Find $x(1)\,,\ y(1)$ by taking $\Delta t = \frac{1}{3}$.

9. Find the general solution of the system of differential equations

$$\frac{dx}{dt} = 3x$$
$$\frac{dy}{dt} = x^2 - 2y,$$

with the initial conditions $x(0) = 1,\ y(0) = 2$.

10. Solve the system of linear equations $\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, with the initial conditions $x(0) = \frac{1}{2}$, $y(0) = 1$.

11. At $t = 0$ a current of 2 amperes flows in an RLC circuit governed by

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t).$$

Assume resistance $R = 10$ ohms, inductance $L = 0.5$ henrys, and capacitance $C = 10^{-2}$ farads. Find the current flowing in the circuit at $t > 0$, if the initial charge on the capacitor is 1 coulomb. Assume that $E(t) = 0$ for $t > 0$.

12. Consider the undamped harmonic oscillator with the external force is governed by $\frac{d^2y}{dt^2} + 4y = \cos wt$, $y(0) = 0$, $y'(0) = 1$. Find the position of the oscillator when the system is in resonance.

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