



Department of Mathematics
School of Advanced Sciences
Fall Semester 2023-2024
Continuous Assessment Test-I

Slot: C1+TC1+TCC1

Course Code: BMAT203L

Course Title: Linear Algebra and Differential Equations

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Class Number(s): 2190,2191

Programme Name & Branch: B.Tech & BBT

Exam Duration: 90 Min

Answer all the questions. Each question carries 10 marks.

1. Write the augmented matrix for the following system of equations and reduce it into RREF. Check whether the system is consistent, if it is consistent write the solution set.

RREF

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$\ell(A) = \ell(A/B) \rightarrow$ consistent; rank < no. of variables $\rightarrow$ infinite solns.

$x_4 = 2$ (free variable)

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 0 \\ x_1 + x_2 + 2x_3 - x_4 &= 1 \\ x_1 - x_2 + 2x_3 + x_4 &= 0 \end{aligned}$$

$x_1, x_2, x_3 \rightarrow$ basic

$$\begin{aligned} x_3 &= 1 \\ x_2 &= 1/2 + 2 \\ x_1 &= -3/2 \end{aligned}$$

2. Find eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$.

3. (a) Show that the sequence $a_n = \frac{3n+2}{4n+3}$ is bounded.

$a_n < 0$ and $\lim_{n \rightarrow \infty} a_n = 3/4 \rightarrow$ bounded

$a_{n+1} > a_n$

- (b) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{9n+5}$ is absolutely convergent, conditionally convergent, or divergent; justify your answer.

check using comparison test, all series if it is diverging $\Rightarrow$ conditionally conv.

4. Discuss the convergence of the following series

(a) $\sum_{n=0}^{\infty} \frac{n^2+2}{n^2+1} \rightarrow$ use ratio converges

[4M]

(b) $\frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \dots \rightarrow$ ratio or root test

diverges

[6M]

5. Find the third degree Taylor polynomial of the function $f(x) = x \sin(x)$ centered at π .

$$f(x) = 0 + \frac{(-\pi)}{1!}(x-\pi) + \frac{(-2)}{2!}(x-\pi)^2 + \frac{(+\pi)}{3!}(x-\pi)^3$$