



KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

- Consider the following system of linear equations
 $x + y + z + 2a w = 2$; $2y + 2z - a w = 1$ and $x + y - z + a^2 w = b$.
 Find the conditions on the parameters a and b such that the system will have
 (i) a unique solution (ii) infinitely many solutions (iii) no solution.
- Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & -1 \\ 3 & 1 & 1 \end{pmatrix}$.
- a) Show that the sequence $a_1 = 3, a_{n+1} = \frac{a_n}{2}$ is decreasing and bounded. [5]
 And hence convergent.
 b) Check the convergence/divergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n+2}$. Justify your response. [5]
- Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(2x-4)^n}{3^n}$.
- Find the fourth degree Taylor series of the function $f(x) = e^x \cos x$ about the point $a = \pi$.
- A small metal bar, whose initial temperature was 20°C , is dropped in to large container of boiling water 100°C . How long will it take the bar to reach 90°C if it is known that its temperature increase 2°C per second?
- a) Draw the slope field of the differential equation $\frac{dy}{dt} = y + t^2$. [5]
 b) Find the general solution of the differential equation $\frac{dy}{dt} = y t$, $y(0) = 1$. [5]
- Find the general solution of the system of differential equations
 $\frac{dy}{dt} = -4y$; $\frac{dx}{dt} = 3x - y^2$, with $y(0) = 4$, $x(0) = 2$.
- Consider the following system of first order differential equations
 $\frac{dx}{dt} = y$; $\frac{dy}{dt} = -\sin x$. Assume $x(0) = 0$, $y(0) = 2$, $\Delta t = \frac{\pi}{4}$, find $x(\pi)$, $y(\pi)$.
- Formulate the governing equation of the damped harmonic oscillator with mass $m = 1$, spring constant $k = 8$, and the damping coefficient $c = 6$. Find the oscillator position at any time t , when the initial displacement is zero and initial velocity is 1. Also, classify the oscillator.
- Using matrix method solve the system $\frac{dx}{dt} = 3x - 10y$; $\frac{dy}{dt} = x - 3y$, with $x(0) = 0$, $y(0) = 1$.
- Find the displacement at any time t , of undamped harmonic oscillator with sinusoidal forcing given by $\frac{d^2y}{dt^2} + 9y = 5 \sin t$, with $y(0) = 0$, $y'(0) = 1$.