VIT
Vellore Institute of Technology

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test – I

Programme Name & Branch : **B.Tech**

Slot: **D1+TD1**

Course Name & code: **Probability and Statistics & BMAT202L**

Class Number (s): VL2023240501672/1745/1670/2282/2299/1664/1661

Exam Duration: 90 Min. **Maximum Marks: 50**

Answer ALL Questions

(Only calculator is to be permitted)

Q.No.	Question	Max Marks
1.	Calculate the missing frequency X and median for the following data: No. of pills | No. of people cured 4 – 8 | 11 8 – 12 | 13 12 – 16 | 16 16 – 20 | 14 20 – 24 | X 24 – 28 | 9 28 – 32 | 17 32 – 36 | 6 36 – 40 | 4 Given that the average number of pills to cure a person is 20.	10

2\. Calculate the Quartile deviation and the Standard deviation of the number of children in 35 families for the following data:

No. of children	0	1	2	3	4	5
No of families	2	3	10	15	4	1

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3\. A random variable X has probability density function

$$f(x) = \begin{cases} kx^2 e^{-3x} & , \ x > 0 \\ 0 & , \ x \le 0 \end{cases}$$

Find (i) the constant k

(ii) $P(1 < X < 2)$

(iii) $P(X \geq 3)$

(iv) $P(X < 1)$

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4\. The joint probability mass function of two random variables X and Y is specified as follows,

$$P[X = x, Y = y] = k\,(2x + 3y), \qquad x = 1,2\,; y = 0,1,2.$$

Obtain (i) the constant k.

(ii) the marginal probability distribution of X.

(iii) the conditional probability of Y given X = 1.

(iv) $P(X > 1, Y < 2)$.

(v) Check whether X and Y are independent or not.

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5\. A sample of 10 fathers and their sons gave the following data about their heights in inches:

Father X	65	63	67	64	68	62	70	66	68	67
Son Y	68	66	68	65	69	66	68	65	71	67

Calculate the correlation coefficient of X and Y and comment on the result.

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CAT - I BMAT202L (P & S)

(1) Given $\bar{X} = 20 = \frac{\sum_i f_i x_i}{\sum_i f_i}$

$\sum f_i x_i = 1772 + 22X$

$\sum f_i = 90 + X$

$\therefore \bar{X} = \frac{1772 + 22X}{90 + X} = 20$

$\Rightarrow \boxed{X = 14}$ (i.e) Missing frequency

Median class : 16-20

Median $= 16 + \left[\frac{52 - 40}{14}\right] \times 4$

$= 19.4286.$

(2) $N = \sum_i f_i = 35$

$N/4 = 8.75$; $\frac{3N}{4} = 26.25$

$Q_1 = 2$; $Q_3 = 2$

$Q.D. = \frac{Q_3 - Q_1}{2} = 0.5.$

$\sum_i f_i x_i^2 = 267.$ $\sum f_i x_i = 89$

$S.D. = \sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2}$

$= \sqrt{7.6286 - (2.5429)^2}$

$= \sqrt{1.1623} = 1.0781.$

(3) As $f(x)$ is a p.d.f, $\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^{\infty} k x^2 e^{-3x}\,dx = 1 \Rightarrow k = 27/2.$

(ii) $P(1 < X < 2) = \frac{35}{2}(e^{-3} - e^{-6})$

$= \frac{35(e^3 - 1)}{2e^6}.$

(iii) $P(X \geq 3) = \frac{101}{2e^9}.$

(iv) $P(X > 1) = 1 - \frac{17}{2e} = \frac{2e - 17}{2e}.$

(4) $P(X = x, Y = y) = k(2x + 3y)$, $x = 1, 2$; $y = 0, 1, 2$

X \ Y	0	1	2	P_{i*}
1	2k	5k	8k	15k
2	4k	7k	10k	21k
P_{*j}	6k	12k	18k	1

$\sum_{i=1}^{2} \sum_{j=1}^{3} P_{ij} = 1 \Rightarrow k = 1/36.$

(ii) Marginal prob. distn. of X:

X	1	2
P_{i*}	15/36	21/36

(iii) Condl. prob. fn. of Y gn. X=1.

$P(Y=0 \mid X=1) = 2/15$; $P(Y=2 \mid X=1) = \frac{8}{15}$

$P(Y=1 \mid X=1) = 5/15$

(iv) $P(X > 1, Y < 2) = 4k + 7k = 11/36.$

(v) X & Y are not independent.

(5) $\sum X = 660$; $\sum X^2 = 43616$

$\sum Y = 673$; $\sum Y^2 = 45325$

$\sum XY = 44445$; $n = 10$

$r_{XY} = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{\{n(\sum X^2) - (\sum X)^2\}\{n\sum Y^2 - (\sum Y)^2\}}}$

$0.6368 = \frac{270}{423.9811} = \frac{270}{\sqrt{560 \times 321}}$

There is a positive correlation.

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