VIT®
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test – I

Programme Name & Branch: B.Tech

Slot: D2+TD2

Course Name & code: Probability and Statistics- BMAT202L

Class Number (s): VL2023240501665, VL2023240502271, VL2023240502291, VL2023240501744, VL2023240501662, VL2023240502275, VL2023240502278.

Faculty Name (s): MURUGAN V , GOURANGA MALLIK, PADMA R, DEBAROTI DAS, POORNIMA T, RAMU G, DHARANI S.

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Answer all questions 5×10=50

Q.No.	Question	Max Marks	CO	BL
1.	Calculate the mean, median and mode for the following distribution. Marks: 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 No. of students: 8 | 87 | 190 | 304 | 211 | 85 | 20	10	CO1	BL3
2.	Find the coefficient of mean deviation from mean, coefficient of variation for the following data. x: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 f: 4 | 36 | 100 | 232 | 280 | 204 | 112 | 28 | 4	10	CO1	BL3
3.	Let X and Y be two random variables having the joint probability mass function $f(x,y)=\frac{1}{27}(2x+y)$ where x and y can assume only the integer values 0, 1, 2. (i) Find all marginal distributions and means of X and Y. (ii) Determine the value of $P[X\le 1/Y=1]$ and $P[X\ge 1, Y<2]$	10	CO2	BL3

4.	Let X and Y have the joint probability density function $f(x,y)=\begin{cases} x^2+\frac{xy}{3}, 0\le x\le 1,\ 0\le y\le 2 \\ 0, \quad otherwise \end{cases}$ Then find (i) $P\left(X>\frac{1}{2}\right)$ (ii) $P(Y<X)$ (iii) $P\left(Y<\frac{1}{2}/X<\frac{1}{2}\right)$	10	CO2	BL3
5.	Calculate the Karl-Pearson's coefficient of correlation for the following percentage of marks in Economics (E) and Statistics (S) S.No 1 2 3 4 5 6 7 8 9 10 E 78 36 98 25 75 82 90 62 65 39 S 84 51 91 60 68 62 86 58 53 47	10	CO3	BL2

CAT - I

D_2 - Slot Key

BMAT 202L — Probability & Statistics

(1)

Marks	f	Mid (x)	xf	Cum. fr
29.5 – 39.5	8	34.5	276	8
39.5 – 49.5	87	44.5	3871.5	95
49.5 – 59.5	190	54.5	10355	285
59.5 – 69.5	304	64.5	19608	589
69.5 – 79.5	211	74.5	15719.5	800
79.5 – 89.5	85	84.5	7182.5	885
89.5 – 99.5	20	94.5	1890	905
	$\Sigma f = 905$		58902.5	

→ Mean $\frac{\Sigma fx}{\Sigma f} = 65.0856$

→ Median Class $\frac{N}{2} = 452.5$

Class 59.5 – 69.5

$L = 59.5$, $f = 304$, $C.f = 285$, $i = 10$

$$\text{Median} = L + \left(\frac{N}{2} - C.f\right)\frac{i}{f} = 65.0099$$

→ Modal Class 59.5 – 69.5

$L = 59.5$, $f_1 = 304$, $f_0 = 190$, $f_2 = 211$

$i = 10$

$$\text{Mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 65.0072$$

(2) Mean $= \frac{\sum fx}{N} = \frac{3972}{1000} = 3.972$

x	f	$D = \lvert x - \bar{x} \rvert$	fD	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	4	3.972	15.88	15.776	63.104
1	36	2.972	106.992	8.882	317.952
2	100	1.972	197.2	3.888	388.8
3	232	0.972	225.504	0.944	219.008
4	280	0.028	7.84	0.00078	0.2184
5	204	1.028	209.712	1.056	215.424
6	112	2.028	227.136	4.112	460.544
7	28	3.028	84.784	9.168	256.704
8	4	4.028	16.112	16.224	64.896

Mean deviation $= \frac{\sum f\lvert D \rvert}{N} = \frac{1091.168}{1000}$

$= 1.091168$

Coefficient of Mean deviation $= \frac{M.D}{\text{Mean}} = \frac{1.091168}{3.972}$

$= 0.2747$

Standard deviation $= \sqrt{\frac{\sum f(x-\bar{x})^2}{N}} = \sqrt{\frac{1986.6504}{1000}}$

$= 1.4094$

Coefficient of S.D $= \frac{S.D}{\text{mean}} = 0.35498$

Coefficient of variation $= 100 \times \frac{S.D}{\text{mean}} = 35.498$

(3)

Marginal distribution of X,

$$P_X(x) = \sum_y P(x,y) = P(x,0)+P(x,1)+P(x,2)$$

$$P_X(x) = \begin{cases} \frac{1}{9} & \text{if } x=0 \\ \frac{1}{3} & \text{if } x=1 \\ \frac{5}{9} & \text{if } x=2 \end{cases}$$

$$P_Y(y) = \begin{cases} \frac{2}{9} & \text{if } y=0 \\ \frac{1}{3} & \text{if } y=1 \\ \frac{4}{9} & \text{if } y=2 \end{cases}$$

Mean $E(X) = \sum x\, P(x,y) = \frac{13}{9}$

$$E(Y) = \sum y\, P(x,y) = \frac{11}{9}$$

$$P(X \le 1 \mid Y=1) = \frac{P(X,Y)}{P_Y(Y)} = \frac{P(X \le 1, 1)}{P_Y(1)}$$

$$= \frac{P(0,1)+P(1,1)}{P_Y(1)}$$

$$= 4/9$$

$$P(X \ge 1, Y < 2) = P(1,0)+P(2,0)+P(1,1)+P(2,1)$$

$$= \frac{2}{27}+\frac{4}{27}+\frac{3}{27}+\frac{5}{27} = \frac{14}{27}$$

(4) (i) $P(X > \frac{1}{2}) = \int_{x=1/2}^{1} \int_{y=0}^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= 5/6$

(ii) $P(Y < X) = \int_{x=0}^{1} \int_{y=0}^{x} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= \frac{7}{24}$

(iii) $P\left(Y < 1/2 \mid X < 1/2\right) = \frac{P(Y < 1/2) \cap P(X < 1/2)}{P(X < 1/2)}$

$P(Y < 1/2) \cap P(X < 1/2) = \int_{x=0}^{1/2} \int_{y=0}^{1/2} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= \frac{5}{192}$

$P(X < 1/2) = \int_{0}^{1/2} \int_{0}^{2} \left(x^2 + \frac{xy}{3}\right) dy\, dx$

$= 1/6$

$\therefore \quad P\left(Y < \frac{1}{2} \mid X < \frac{1}{2}\right) = \frac{5}{32}$

(5)

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
78	84	13	18	169	324	234
36	51	-29	-15	841	225	435
98	91	33	25	1089	625	825
25	60	-40	-6	1600	36	240
75	68	10	2	100	4	20
82	62	17	-4	289	16	-68
90	86	25	20	625	400	500
62	58	-3	-8	9	64	24
65	53	0	-13	0	169	0
39	47	-26	-19	676	361	494
			Total =	5938	2224	2704

$$\bar{x} = \frac{\sum x}{n} = \frac{650}{10} = 65$$

$$\bar{y} = \frac{\sum y}{n} = \frac{660}{10} = 66$$

$$r = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2} \sqrt{\sum (y-\bar{y})^2}}$$

$$= \frac{2704}{73.47 \times 47.16}$$

$$= 0.7804$$

+ Vely Correlated.