Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES

Winter Semester 2023-2024

Continuous Assessment Test –II

Programme Name & Branch : **B.Tech**

Slot : **D1+TD1 (Common)**

Course Name & Code : **Probability and Statistics & BMAT202L**

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): Answer ALL Questions

(Statistical Table is to be permitted)

Q.No	Question	Max Marks	CO	BL
1.	Obtain the equations of the regression lines from the following data. Also estimate the values of (i) Y, when X=38 and (ii) values of X, when Y=18.	10	CO3	BL3
2.	In a certain factory producing razor blades, there is a small chance 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective blade, (ii) at least one defective blade and (iii) at most 1 defective blade in a consignment of 10000 packets.	10	CO2	BL5
3.	The lifetime X (in hundreds of hours) of a certain type of vacuum tube has a Weibull distribution with parameters $\alpha=2$ and $\beta=3$. Compute the following: (i) E(X) and Var(X), (ii) $P(X\leq 6)$, (iii) $P(1.8\leq X\leq 6)$ and (iv)$P(X\geq 3)$.	10	CO2	BL2
4.	A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years at 0.05 level of significance.	10	CO4	BL5

X	22	26	29	30	31	31	34	35
Y	20	20	21	29	27	24	27	31

5.	In a referendum submitted to the student body at a university, 850 men and 566 women voted. 530 of the men and 304 of the women votes yes. Does this indicate a significant difference of the opinion on the matter between men and women students?	10	CO4	BL5

Answer key D1+TTD1

BMAT202L – Probability and Statistics

① $y = 0.83x + 0.18$, $r = 0.82$ $\quad y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$x = 0.81y + 9.60$ $\quad x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

If $X = 38$, $Y = 31.72$

If $Y = 18$, $X = 24.18$

② $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$ $\lambda > 0$.

$\lambda = 0.02$ $\quad p = \frac{1}{500}$ $\quad n = 10$

$10{,}000 \times P(X = 0) \simeq 9802$ $\quad P(X = 0) = e^{-0.02} = 0.9802$

$10{,}000 \times P(X \geq 1) \simeq 198$ $\quad P(X \geq 1) = 0.0198$

$10{,}000 \times P(X \leq 1) \simeq 10{,}000$ $\quad P(X \leq 1) \simeq 1$

③ $\alpha = 2$, $\beta = 3$. Pdf of X is $f(x; \alpha, \beta) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$, $x > 0$, $\alpha, \beta > 0$.

$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$

(i) $E(X) = 2.6587$, $V(X) = 1.9318$

(ii) $P(X \leq 6) = 0.9817$ (iii) $P(1.8 \leq X \leq 6) = 0.6794$

(iv) $P(X \geq 3) = 0.3679$

④ $H_0 : \mu = 70$ yrs $\quad \alpha = 0.05$ $\quad \bar{x} = 71.8$ $\quad Z_\alpha \geq 1.645$

$H_1 : \mu > 70$ yrs $\quad Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ $\quad \sigma = 8.9$

$= 2.02$

Reject H_0 and conclude that the mean life span today is greater than 70 yrs.

⑤ $p_1 = \frac{530}{850} = 0.62$ $\quad p_2 = \frac{304}{586} = 0.54$

i.e. $Z_\alpha = 1.58$ $\quad p = 0.59$ $\quad q = 0.41$

$Z = 3.02$. $\quad |Z| > Z_\alpha$, the difference b/w p_1 and p_2 is significant, viz the men and the women students have significantly different opinions on the matter.