School of Computer Science and Engineering

Winter Semester 2023-24

Continuous Assessment Test – 1

SLOT: B1+TB1

Programme Name &Branch: B.Tech

Course Name & code: BCSE304L Theory of Computation

Class Number (s): VL2023240500758, 0762, 0764, 0767, 0769, 0770, 0773, 0783, 0788, 0794, 0842, 0859, 1011, 1013, 1024, 1027, 1028, 1031, 1034, 1038, 1040

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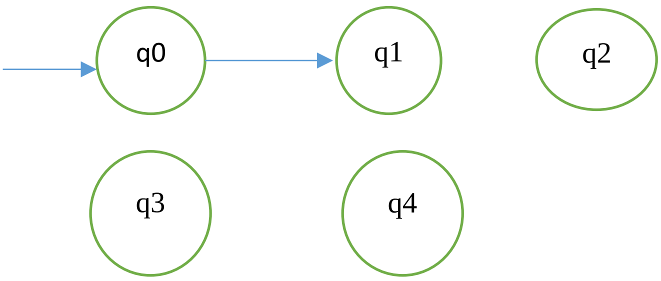
Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s): - Step by Step Procedure is required to solve the Problem

<table>
<tr><th>Q.No.</th><th>Question</th><th>Max Marks</th><th>CO</th><th>BL</th></tr>
<tr><td>1.</td><td>a. Prove using mathematical induction for the following
|uv|= |u|+ |v| for u,v are strings over Σ. (3 Marks)
b. Consider A={00,10,20}, ∅ ={ } , L={ ε } and Σ = {0,1,2}.
Then compute the following
1. L^*
2. $\emptyset^*$
3. Σ^2-A where – is a setminus operation. (4 Marks)
c. Give an example for
(i) L & L^c (c is a complementary operation of L) are infinite.
(ii) L is finite and L^c is infinite. (3 Marks)</td><td>10</td><td>CO1</td><td>BL2</td></tr>
<tr><td>2.</td><td>Convert the following NFA with ε to NFA without ε.
<table><tr><th></th><th>a</th><th>b</th><th>c</th><th>ε</th></tr><tr><td>-> q0</td><td>q0</td><td>∅</td><td>∅</td><td>q1</td></tr><tr><td>q1</td><td>∅</td><td>q1</td><td>∅</td><td>q2</td></tr><tr><td>* q2</td><td>∅</td><td>∅</td><td>q2</td><td>∅</td></tr></table>Starting state : q0
Final State : q2</td><td>10</td><td>CO2</td><td>BL3</td></tr>
</table>

3. a) Construct a DFA which accepts all strings over the alphabet $\Sigma = \{0,1\}$ which can be seen as a binary representation of numbers that are divisible by 4. For example, number 5 corresponds to the string 101 (which is a binary number representation for 5) and the string 101 should be rejected by the DFA. The following is an incomplete attempt to construct the automata for the above language. Complete it [find transitions and final state(s)].



(5 Marks)

b) Convert the following NFA to DFA.

	a	b
-> q0	{q0,q1}	q0
q1	q2	q2
*q2	∅	∅

Starting state : q0
Final State : q2 (5 Marks)

10 CO2 BL3

4. i) Minimize the DFA whose transition table is given below.

	0	1
-> q0	q1	q3
q1	q2	q4
q2	q3	q2
*q3	q4	q5
*q4	q3	q4
*q5	q4	q5

Starting state : q0
Final State : {q3,q4,q5} (5Marks)

ii) Given a regular expression for the Language that contains **ab** or **ba** in the sting for $\Sigma = \{a,b\}$ (2 Marks)

iii) Convert the following Regular Expression to Finite Automata. **(a*+bc)** (3 Marks)

10 CO2 BL3

5. Convert the given Finite Automata transition table to Regular Expression.

	a	b
-> q0	q1	q1
q1	q2	{q1,q3}
q2	∅	q3

10 CO2 BL3

	| *q3 | q3 | ∅ | Start State: q0 Final State: q3			

NOTE*: Please refer below to the BL – Bloom's Taxonomy Levels and mention the respective level in the questions.

Bloom's Taxonomy Levels	**Category**
BL1	Remembering
BL2	Understanding
BL3	Applying
BL4	Analyzing
BL5	Evaluating
BL6	Creating

Q.(1) a) w is length of the string, denoted by $|w|$

Note that $|\epsilon| = 0$

If u & v are two strings, then

$$|uv| = |u| + |v|$$

Let us define the length of the string recursively by

$$|a| = 1$$

$\therefore$ $|wa| = |w| + 1$ $\forall\, a \in \Sigma$ and w any string on Σ. ← ①

take any v of length $n+1$ and write it as

$$v = wa$$

$\therefore$ $|v| = |w| + 1$ from equ ①

$|uv| = |uwa|$

$= |uw| + 1$

By induction hypothesis

$$|uw| = |u| + |w|$$

so that

$$|uv| = |u| + |w| + 1$$

$$= |u| + |v|$$

(3 Marks)

1. b)

(i) 1) $L^* = \{\epsilon\}$

2) $\phi^* = \{\epsilon\}$

3) $\Sigma^2 - A = \{$

$\Sigma^2 = \{00, 01, 02, 10, 12, 11, 20, 21, 22\}$

$A = \{00, 10, 20\}$

$\therefore$ $\Sigma^2 - A = \{01, 02, 12, 11, 21, 22\}$

(4 Marks)

1) c) $L = \{$set of all even length over $\{0,1\}\}$

(i) $\therefore$ $L^c = \{$set of all odd length over $\{0,1\}\}$

$L = \{aa, ab, ba, bb, aaba, \ldots\}$ $L^c = \{a, b, aaa, aab, \ldots\}$

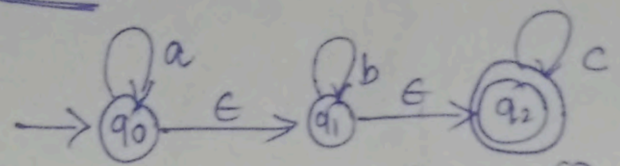
(ii) $L = \{a\}$ over $\{a\}$

L^c is infinite.

(3 Marks)

...A-ε to NFA

Q.(2) (10 Marks)



ϵ-closure$(q_0) = \{q_0, q_1, q_2\}$ —— (1)
ϵ-closure$(q_1) = \{q_1, q_2\}$ —— (2)
ϵ-closure$(q_2) = \{q_2\}$ —— (3)

$\delta'(q_0, a) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), a))$
$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), a)$
$= \epsilon\text{-closure}(q_0)$
$= \{q_0, q_1, q_2\}$ —— (I)

|||rly $\delta'(q_0,a)$

$\delta'(q_0, b) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0)), b)$
$= \epsilon\text{-closure}(\delta(q_0, q_1, q_2), b)$
$= \epsilon\text{-closure}(q_1)$
$= \{q_1, q_2\}$ —— (II)

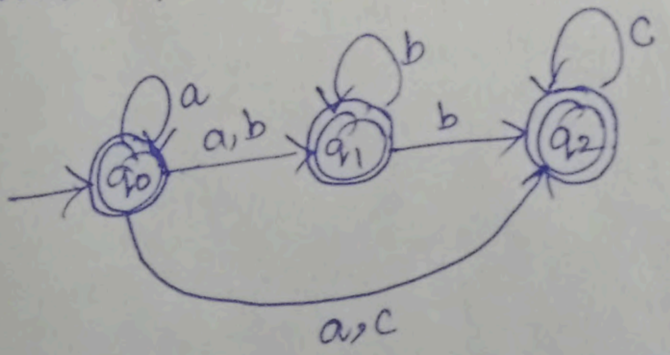
$\delta'(q_0, c) = \epsilon\text{-closure}(q_2)$
$= q_2$ —— (III)

|||rly find

$\delta'(q_1, a) = \phi$ —— (IV)
$\delta'(q_1, b) = \{q_1, q_2\}$ —— (V)
$\delta'(q_1, c) = \phi$ —— (VI)

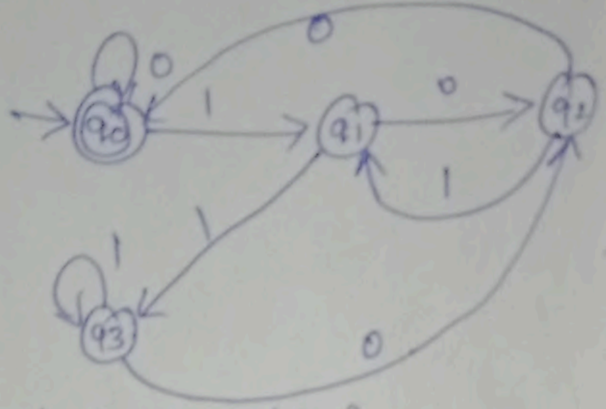
$\delta'(q_2, a) = \phi$ —— (VII)
$\delta'(q_2, b) = \phi$ —— (VIII)
$\delta'(q_2, c) = \{q_2\}$ —— (IX)

from eqn (I) to (IX), construction of NFA without ε



q_2 is final state. It is member in the
$\delta'(q_0, a) = \{q_1, q_2, q_3\}$ ∴ q_0, q_1 are
$\delta'(q_1, b) = \{q_1, q_2\}$ also final state.

Q.(3)(a) construct DFA, which accepts all string over an alphabets Σ which is divisible by 4



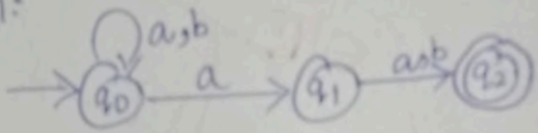
dec	Bin	Reminder
0	0	0
1	01	1
2	10	2
3	11	3
4	100	0
.	.	
.	.	
8	1000	0

Final state = $\{q_0\}$
from the ~~inputs~~ incomplete diagram. q_4 is not required.

(5 marks)

Q(b) (3) NFA to DFA

sol:



Table

	a	b
q_0	$\{q_0,q_1\}$	q_0
q_1	q_2	q_2
q_2	ϕ	ϕ

$\hat{\delta}(q_0, a) = q_0q_1$ ——— ① New state.

$\hat{\delta}(q_0, b) = q_0$

$\hat{\delta}(\{q_0q_1\}, a) = \delta(q_0, a) \cup \delta(q_1, a)$
$= \{q_0, q_1\} \cup \{q_2\}$
$= \{q_0, q_1, q_2\}$ ——— ②

$\hat{\delta}(\{q_0, q_1\}, b) = \{q_0, q_2\}$ ——— ③

$\hat{\delta}(\{q_0, q_1, q_2\}, a) = \{q_0, q_1, q_2\}$

$\hat{\delta}(\{q_0, q_1, q_2\}, b) = \{q_0, q_2\}$

$\hat{\delta}(\{q_0, q_2\}, a) = \{q_0, q_1\}$

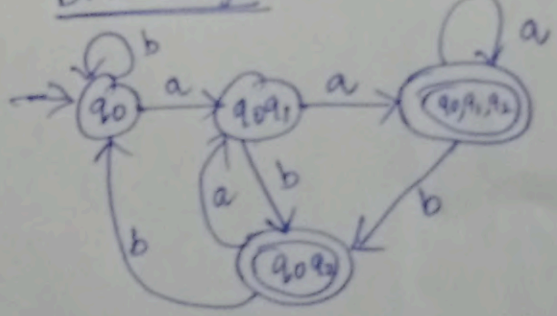
$\hat{\delta}(\{q_0, q_2\}, b) = q_0$

$\therefore$ 3 New states

DFA Transition Table:

	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
$* \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
$* \{q_0, q_2\}$	$\{q_0, q_1\}$	q_0

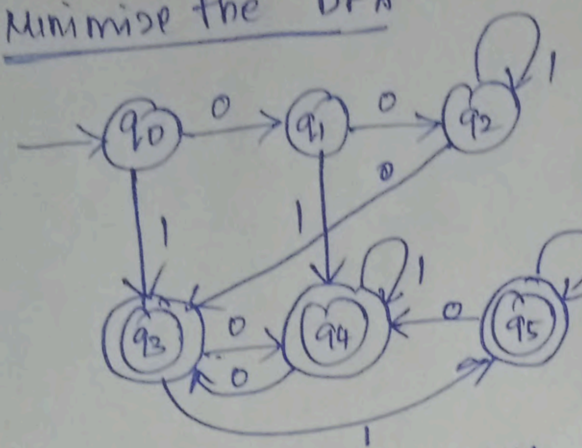
DFA Diagram



(5 marks)

Q (1) Minimise the DFA

(i)



(5 Marks)

by using Table filling method.

	q_0	q_1	q_2	q_3	q_4	q_5
q_0						
q_1	✓					
q_2	✓	✓				
q_3	✓	✓	✓			
q_4	✓	✓	✓	.		
q_5	✓	✓	✓	.	.	

(q_1, q_0)

$\delta(q_1,0) = q_2$ $\quad \delta(q_1,1) = q_4$

$\delta(q_0,0) = q_1$ $\quad \delta(q_0,1) = q_3$

(q_2, q_1)

$\delta(q_2,0) = q_3$ is

$\delta(q_1,0) = q_2$ marked

$\therefore (q_2, q_1)$ is ✓

(q_2, q_0) $\quad \delta(q_2,0) = q_3$ is marked

$\delta(q_0,0) = q_1$ ✓.

$\therefore (q_2, q_0)$ also mark

(q_4, q_3)

$\delta(q_4,0) = q_3$ $\quad \delta(q_4,1) = q_4$

$\delta(q_3,0) = q_4$ $\quad \delta(q_3,1) = q_5$

(q_5, q_3)

$\delta(q_5,0) = q_4$ × $\quad \delta(q_5,1) = q_5$ ×

$\delta(q_3,0) = q_4$ $\quad \delta(q_3,1) = q_5$

$\delta(q_5, q_4)$

$\delta(q_5,0) = q_4$ $\quad \delta(q_5,1) = q_5$

$\delta(q_4,0) = q_3$ $\quad \delta(q_4,1) = q_4$

repeat this step until no marking to be done.

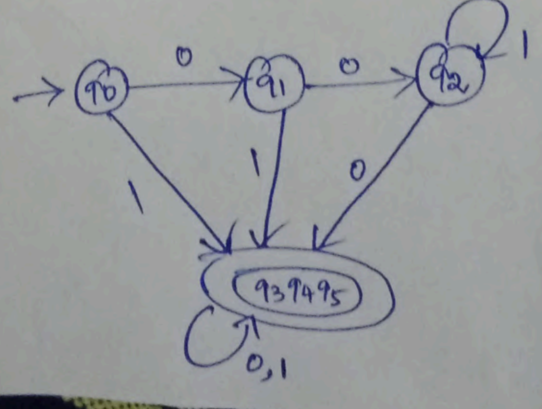
unmarked pair

(q_3, q_4) (q_3, q_5) (q_4, q_5).

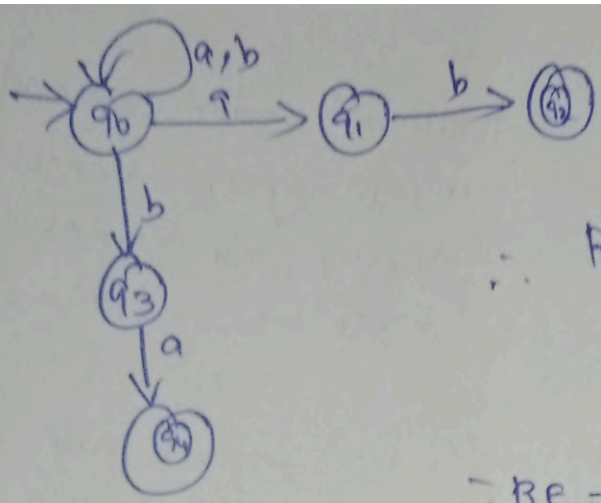
(q_3, q_4, q_5) (q_4, q_5)

(q_3, q_4, q_5)

$\therefore$ Minimized DFA is



Q 4) (ii)



∴ Regular expression for the Language that contains ab or ba.

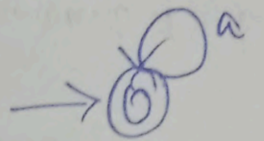
$\therefore RE = (a+b)^* (ab+ba)$

(2 Marks)

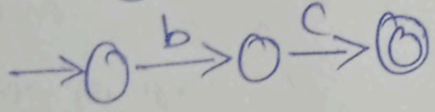
Q) 4 (iii) RE to FA

$a^* + bc$

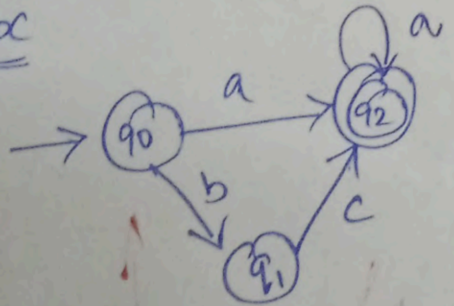
a^*



bc



∴ $a^* + bc$

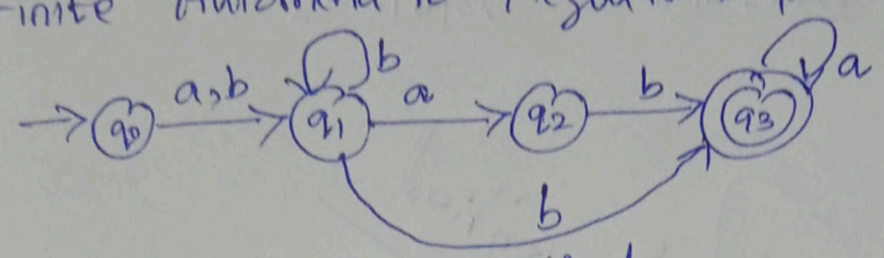


$\therefore M = \{ Q = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \delta, q_0, \{q_2\} \}$.

(3 Marks)

(5) Finite Automata to Regular Expression (10 marks)

Sol:



By using Arden's method

$q_0 = \epsilon$ —— (1)

$q_1 = q_0 a + q_1 \cdot b$ —— (2)

$q_2 = q_1 \cdot a$ —— (3)

$q_3 = q_1 \cdot b + q_2 \cdot b + q_3 \cdot a$ —— (4)

write the final state in the form $R = Q + RP$ then.

RE is $R = QP^*$

$\therefore q_1 = q_0 \cdot a + q_1 \cdot b$

$q_0 = \epsilon$ based on (1)

$q_1 = \epsilon \cdot a + q_1 \cdot b$

$q_1 = a + q_1 \cdot b$

$R = Q + RP$

$\therefore \boxed{q_1 = ab^*}$

$q_2 = q_1 \cdot a$

$q_2 = ab^* \cdot a$

$q_3 = q_1 \cdot b + q_2 \cdot b + q_3 \cdot a$

$\therefore = ab^* \cdot b + ab^* \cdot a \cdot b + q_3 \cdot a$

$= ab^*(b + a \cdot b) + q_3 \cdot a$

$q_3 = ab^*(b + ab) + q_3 \cdot a$

$R = Q + RP$ by using Ardens method

$q_3 = ab^*(b + ab) \cdot a^*$

$\therefore$ Regular Expression for given Finite Automata is

$\boxed{ab^*(b + ab) \cdot a^*}$