SCHOOL OF ADVANCED SCIENCES

Fall Semester 2024-2025

Continuous Assessment Test – I

Programme Name & Branch: B.Tech

Slot: C1+tC1+TCC1

Course Name & code: Complex Variables and Linear Algebra -BMAT201L

Exam Duration: 90 Min. **Maximum Marks: 50**

Answer ALL the questions,

Q.No.	Question	Max Marks
1.	(i) Show that the function $f(z)=\sqrt{\lvert xy\rvert}$ is not analytic at the origin although Cauchy-Reimann equations are satisfied at the origin. (ii) Find all values of k such that $f(z)=e^{x}(\cos ky+i\sin ky)$ is analytic.	5 5
2.	If $w=f(z)=\phi+i\psi$ represents the complex potential function for an electric field and $\psi(x,\ y)=x^2-y^2+\dfrac{x}{x^2+y^2}$, then find $\phi(x,\ y)$.	10
3.	Find (i) the image of the infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ and (ii) the image of the circle $\lvert z-1\rvert=1$ in the z-plane under the transformation $w=\dfrac{1}{z}$.	10
4.	Find the bilinear transformation which maps the points (-1, 0, 1) into the points (0, i , 3i). Also find the invariant points of it.	10
5.	Obtain the Laurent series expansion of the function $f(z)=\dfrac{7z-2}{(z+1)z(z-2)}$ in the region $1<\lvert z+1\rvert<3$.	10

BMAT201L - Complex Variable & Linear Algebra
$C_1 + TC_1 + TTC_1$ - CAT - I - Keys

1 (i) Here $u(x,y) = \sqrt{|xy|}$, $v(x,y) = 0$

At the origin, $\frac{\partial u}{\partial x} = \lim_{x \to 0} \frac{u(x,0) - u(0,0)}{x - 0} = 0$

Similarly $u_y = 0$, $v_x = 0$, $v_y = 0$. Hence C-R equs are satisfied at origin. Now Along the path $mx = y$, we get

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z - 0} = \lim_{x \to 0} \frac{\sqrt{|x \cdot mx|} - 0}{x(1+im)} = \lim_{x \to 0} \frac{\sqrt{|m|}}{1+mi}$$

$\Rightarrow f(z)$ is not analytic at the origin. ($\because$ It depends on 'm')

(ii) By using C-R eqn, $f(z)$ is analytic when $\underline{\underline{K=1}}$

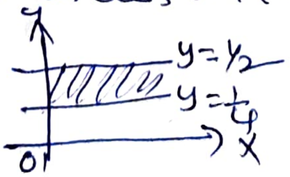
2. By Milne's Thomson's method, $\phi(x,y) = -2xy + \frac{y}{x^2+y^2} + C$

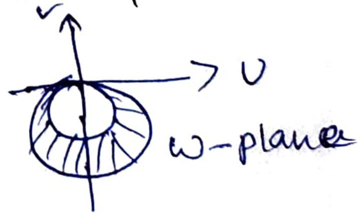
3. (i) $z = \frac{1}{w} \Rightarrow x = \frac{u}{u^2+v^2}$, $y = \frac{-v}{u^2+v^2}$

When $y = \frac{1}{4}$, $\frac{-v}{u^2+v^2} = \frac{1}{4} \Rightarrow u^2 + (v+2)^2 = 4$

When $y = \frac{1}{2}$, $\frac{-v}{u^2+v^2} = \frac{1}{2} \Rightarrow u^2 + (v+1)^2 = 1$

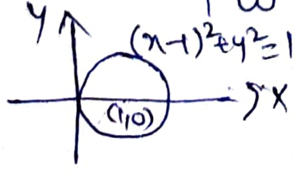
Hence, the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ is transformed into the region in b/t the circles $u^2 + (v+2)^2 = 4$ & $u^2 + (v+1)^2 = 1$ in the w-plane.

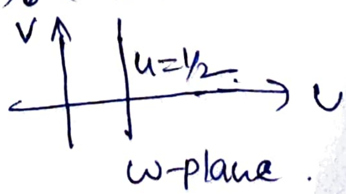




(ii) Given $w = \frac{1}{z}$

Now $|z-1| = 1 \Rightarrow \left|\frac{1}{w} - 1\right| = 1 \Rightarrow \sqrt{(1-u)^2 + (-v)^2} = \sqrt{u^2+v^2} \Rightarrow u = \frac{1}{2}$





4. By the cross ratio of the four points,

We have $w = \dfrac{3i(z+1)}{3-z}$.

The invariant points are $\dfrac{3iz+3i}{3-z}$, $\dfrac{-3(1+i) \pm i\sqrt{30i}}{2}$

5. Given $f(z) = \frac{1}{z} - \frac{3}{z+1} + \frac{2}{z-2}$ (Resolving into partial fractions)

Let $w = z+1 \Rightarrow z = w-1$

For $1 < |z+1| < 3$, $\left|\frac{1}{z+1}\right| < 1$ & $\left|\frac{z+1}{3}\right| < 1$

i.e., $\left|\frac{1}{w}\right| < 1$ & $\left|\frac{w}{3}\right| < 1$

Now $f(z) = \frac{1}{w-1} - \frac{3}{w-1+1} + \frac{2}{w-1-2}$

$= \frac{1}{w}\left(1-\frac{1}{w}\right)^{-1} - \frac{3}{w} - \frac{2}{w}\left(1-\frac{w}{3}\right)^{-1}$

$= \left(-\frac{2}{\cancel{w}} + \frac{1}{w^2} + \frac{1}{w^3} + \cdots\right) - \frac{2}{3}\left(1 + \frac{w}{3} + \frac{w^2}{9} + \cdots\right)$

$= \left(\frac{-2}{z+1} + \frac{1}{(z+1)^2} + \cdots\right) - \frac{2}{3}\left(1 + \frac{z+1}{3} + \cdots\right)$