SCHOOL OF ADVANCED SCIENCES

Fall Semester 2024-2025

Continuous Assessment Test – I

Programme Name & Branch: B.Tech (Common for all the programmes)

Slot: C2+TC2+TCC2

Course Name & code: BMAT201L & Complex Variables and Linear Algebra

Exam Duration: 90 Min. **Maximum Marks: 50**

General instruction(s):Answer All the questions

Q.No.	Question	Max Marks	CO	BL
1.	Show that the function $f(z) = z^2\bar{z}$ is differentiable only at the origin but nowhere analytic.	10	CO1	BL4
2.	If $\omega = \phi + i\psi$ represents the complex potential for an electric field and $\psi = e^{-x}(\,2xy\cos y + (y^2 - x^2)\sin y\,)$ is given. Construct the complex electric potential for the given stream line function ψ.	10	CO1	BL5
3.	Find the image of the triangular region in the z- plane bounded by the lines $x = 0, y = 0$ and $x + y = 2$ under the mapping $\omega = (1 + i)z$.	10	CO2	BL2
4.	Find the bilinear transformation which maps $z = 0,1,\infty$ respectively on to $\omega = -5,-1,3$. Hence find the fixed points.	10	CO2	BL5
5.	Find the Laurent's series expansion of the function $f(z) = \frac{z}{(z-2)(z-3)}$ which are valid in the range (i) $1 < \lvert z-1\rvert < 2$, (ii) $\lvert z-1\rvert > 2$.	10	CO2	BL3

CAT-1 C2 - BMAT201L

Answer key CV & LA

① $f(z) = z^2 \bar{z}$ | $= z \cdot |z|^2$
| $= (x+iy)(x^2+y^2)$

$= (x+iy)^2 \cdot (x-iy)$

$= (x^3+xy^2) + i(y^3+x^2y)$ $\longrightarrow$ 2M

$u = x^3+xy^2$; $v = y^3+x^2y$

$u_x = 3x^2+y^2$ $\quad v_x = 2xy$ $\quad u_x = v_y$

$u_y = 2xy$ $\quad v_y = 3y^2+x^2$ $\quad v_x = -u_y$ only at $(0,0)$

C-R eqn satisfied only at $(0,0)$ $\longrightarrow$ 6M

$\therefore$ Differentiable only at origin. (Dif at z_0 in nbhd

$\Rightarrow$ Nowhere analytic. $\longrightarrow$ 2M

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② $\psi = e^{-x}(2xy\cos y + (y^2-x^2)\sin y)$

$\psi_x(z,0) = 0$, $\quad \psi_y(z,0) = e^{-z}(2z - z^2)$ $\longrightarrow$ 5M

M.T $\quad f(z) = \int [\psi_y(z,0) + i\,\psi_x(z,0)]\,dz$

$= \int e^{-z}(2z - z^2)\,dz$

$= 2z e^{-z} + c$ $\longrightarrow$ 5M

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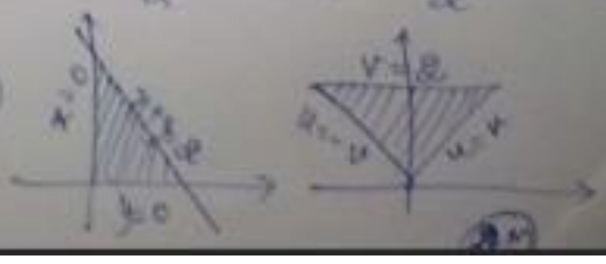
③ $w = (1+i)z$ $\quad u = x-y$; $v = x+y$

$\Rightarrow x = \frac{u+v}{2}$; $y = \frac{v-u}{2}$ $\longrightarrow$ 5M

$x=0 \Rightarrow u=-v$

$y=0 \Rightarrow u=v$

$x+y=2 \Rightarrow v=2$ $\longrightarrow$ 3M



2M

(4) $(0, 1, \infty) \rightarrow (-5, -1, 3)$

$$z = \frac{w+5}{3-w} \Rightarrow \boxed{w = \frac{3z-5}{1+z}} \rightarrow \text{(7M)}$$

Invariant $z = 1 \pm 2i \rightarrow$ (3M)

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(5) $f(z) = \dfrac{z}{(z-2)(z-3)}$ $\quad 1 < |z-1| < 2$

$|z-1| > 2$

$w = z-1$

$$f(w+1) = \frac{w+1}{(w-1)(w-2)} = \frac{-2}{w-1} + \frac{3}{w-2}.$$

(i) $1 < |w| < 2$,

$$f(z) = \frac{-2}{z-1}\left[1 + \frac{1}{z-1} + \frac{1}{(z-1)^2} + \cdots\right]$$

$$- \frac{3}{2}\left[1 + \frac{z-1}{2} + \frac{(z-1)^2}{4} + \cdots\right] \longrightarrow \text{(5M)}$$

(ii) $|w| > 2$,

$$f(z) = \frac{-2}{z-1}\left[1 + \frac{1}{z-1} + \frac{1}{(z-1)^2} + \cdots\right]$$

$$+ \frac{3}{2}\left[1 + \frac{2}{z-1} + \left(\frac{2}{z-1}\right)^2 + \cdots\right] \longrightarrow \text{(5M)}$$